OLYMPIAD CORNER

No. 438

The problems in this section have appeared in a regional or national mathematical Olympiad.

Click here to submit solutions, comments and generalizations to any problem in this section

To facilitate their consideration, solutions should be received by **February 15, 2026**.

OC756. Let $A \in \mathcal{M}_2(\mathbb{R})$ be a matrix with real entries such that

$$\det(A^{2014} - I_2) = \det(A^{2014} + I_2)$$

and

$$\det(A^{2016} - I_2) = \det(A^{2016} + I_2).$$

Prove that $\det(A^n - I_2) = \det(A^n + I_2)$, for any $n \in \mathbb{N}$. Above $\mathbb{N}$ is the set of positive integers and I_2 is the 2×2 identity matrix.

OC757. Find all continuous bijective functions $f : [0, 1] \to [0, 1]$ such that

$$\int_0^1 g(f(x))dx = \int_0^1 g(x)dx,$$

for any continuous function $g : [0, 1] \to \mathbb{R}$.

OC758. Consider $A, B \in \mathcal{M}_n(\mathbb{C})$ such that $AB = BA$ and $\det B \neq 0$.

a) If $|\det(A + zB)| = 1$, for all $z \in \mathbb{C}$ with $|z| = 1$, prove that $A^n = 0_n$;

b) Is the conclusion true if the commutative condition is dropped?

OC759. Let ABC be a scalene triangle, let I be its incentre, and let A_1, B_1, and C_1 be the points of contact of the excircles with the sides BC, CA, and AB, respectively. Prove that the circumcircles of the triangles AIA_1, BIB_1, and CIC_1 have a common point different from I.

OC760. Let $\mathbb{N}$ denote the set of positive integers. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that

$$\gcd(f(x), y)f(xy) = f(x)f(y)$$

for all x and y in $\mathbb{N}$.

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