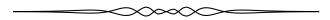
OLYMPIAD CORNER

No. 437

The problems in this section appeared in a regional or national mathematical Olympiad.

Click here to submit solutions, comments and generalizations to any problem in this section

To facilitate their consideration, solutions should be received by January 15, 2026.



OC751. Let $A, B \in \mathcal{M}_n(\mathbb{R})$ be matrices. Consider the matrix function $f: \mathcal{M}_n(\mathbb{C}) \to \mathcal{M}_n(\mathbb{C})$ defined by

$$f(Z) = AZ + B\overline{Z}, \qquad Z \in \mathcal{M}_n(\mathbb{C}),$$

where \overline{Z} is the matrix having as elements the conjugates of the elements of Z. Prove that the following statements are equivalent:

- (i) f is injective;
- (ii) f is surjective;
- (iii) the matrices A + B and A B are invertible.

OC752. Let $f:[0,1] \to \mathbb{R}$ be a continuous function with f(1) = 0. Prove the existence and determine the value of the limit

$$\lim_{\substack{t \to 1 \\ t \to 1}} \left(\frac{1}{1-t} \int_0^1 x \left(f(tx) - f(x) \right) dx \right).$$

OC753. Determine all pairs of prime numbers (p,q) with the following property: there exist positive integers a, b, c satisfying the equality

$$\frac{p}{a} + \frac{p}{b} + \frac{p}{c} = 1, \qquad \frac{a}{p} + \frac{b}{p} + \frac{c}{p} = q + 1.$$

OC754. Solve in real numbers the equation

$$3^{\log_5(5x-10)} - 2 = 5^{-1 + \log_3 x}$$

 $\mathbf{OC755}$. Consider the inscribable pentagon ABCDE in which AB = BC = CD and the centroid of the pentagon coincides with the center of the circumscribed circle. Show that the pentagon ABCDE is regular.