# OLYMPIAD CORNER

## No. 437

*The problems in this section appeared in a regional or national mathematical Olympiad.*

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*To facilitate their consideration, solutions should be received by* **January 15, 2026**.

**OC751**. Let $A, B \in \mathcal{M}_n(\mathbb{R})$ be matrices. Consider the matrix function $f : \mathcal{M}_n(\mathbb{C}) \to \mathcal{M}_n(\mathbb{C})$ defined by

$$f(Z) = AZ + B\overline{Z}, \qquad Z \in \mathcal{M}_n(\mathbb{C}),$$

where $\overline{Z}$ is the matrix having as elements the conjugates of the elements of $Z$. Prove that the following statements are equivalent:

(i) $f$ is injective;

(ii) $f$ is surjective;

(iii) the matrices $A + B$ and $A - B$ are invertible.

**OC752**. Let $f : [0, 1] \to \mathbb{R}$ be a continuous function with $f(1) = 0$. Prove the existence and determine the value of the limit

$$\lim_{\substack{t\to 1\\ t<1}} \left( \frac{1}{1-t} \int_0^1 x \left( f(tx) - f(x) \right) \, dx \right).$$

**OC753**. Determine all pairs of prime numbers $(p, q)$ with the following property: there exist positive integers $a$, $b$, $c$ satisfying the equality

$$\frac{p}{a} + \frac{p}{b} + \frac{p}{c} = 1, \qquad \frac{a}{p} + \frac{b}{p} + \frac{c}{p} = q + 1.$$

**OC754**. Solve in real numbers the equation

$$3^{\log_5(5x-10)} - 2 = 5^{-1+\log_3 x}.$$

**OC755**. Consider the inscribable pentagon $ABCDE$ in which $AB = BC = CD$ and the centroid of the pentagon coincides with the center of the circumscribed circle. Show that the pentagon $ABCDE$ is regular.