OLYMPIAD CORNER

No. 436

The problems featured in this section have appeared in a regional or national mathematical Olympiad.

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To facilitate their consideration, solutions should be received by **December 15, 2025**.

OC746. Two different integers u and v are written on a board. We perform a sequence of steps. At each step, we do one of the following two operations:

(i) If a and b are different integers on the board, then we can write $a+b$ on the board, if it is not already there.

(ii) If a, b and c are three different integers on the board, and if an integer x satisfies $ax^2+bx+c=0$, then we can write x on the board, if it is not already there.

Determine all pairs of starting numbers (u,v) from which any integer can eventually be written on the board after a finite sequence of steps.

OC747. Find all positive integers d for which there exists a degree d polynomial P with real coefficients such that there are at most d different values among $P(0), P(1), P(2), \ldots, P(d^2-d)$.

OC748. The complex $n\times n$ matrices A, B satisfy the relation $A^2B+BA^2=2ABA$. Check that $X=AB-BA$ commutes with A, and either using this or in any other way prove that there exists $k\in\{1,\ldots n\}$ such that $X^k=0$.

OC749. If H is a set containing a given number $n>1$ of (arbitrary) positive integers, how many elements can be in $\{xy+z|x,y,z\in H\}$ at most and at least?

OC750. Find

$$\lim_{n\to\infty}\sum_{k=n}^{2n}\binom{k-1}{n-1}2^{-k}.$$

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