MathemAttic

No. 67

The problems featured in this section are intended for students at the secondary school level.

Click here to submit solutions, comments and generalizations to any problem in this section.

To facilitate their consideration, solutions should be received by **November 15, 2025**.

MA331. *Proposed by Ivan Hadinata.*

Let M be the number of ordered pairs of natural numbers (a, b) satisfying the equation

$$a^b = (20!)^{24!}.$$

Find the last three digits of M.

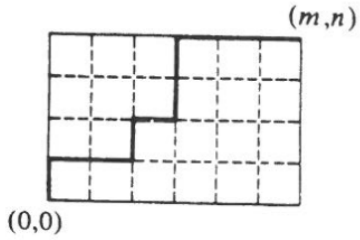
MA332. *Proposed by Michael Friday.*

In any triangle ABC, let H, O be the orthocenter and circumcenter, and let M_a, M_b, M_c be the midpoints of sides BC, CA, AB respectively. Prove that

$$OH^2 = (HM_a^2 - OM_a^2) + (HM_b^2 - OM_b^2) + (HM_c^2 - OM_c^2)$$

MA333.

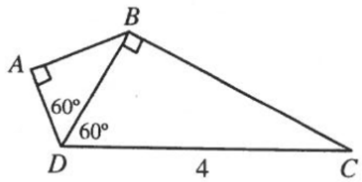
a) An $m \times n$ rectangle is divided into mn squares. A path is to be traced starting at $(0, 0)$ and concluding at (m, n) by moving only in a positive sense along the ruled lines.



Show that the number of distinct paths is $\binom{m+n}{n}$

b) An $n \times n \times n$ cube has each of faces ruled into n^2 squares. A path defined in part a), moving always in a positive sense on its faces, is to start at $(0, 0, 0)$ and reach the point (n, n, n). Determine the number of distinct paths.

MA334. In the quadrilateral $ABCD$, angles DBC and DAB are right angles. Also, angles ADB and BDC have measure of 60 degrees. If DC is 4 units, determine which one is greater $DA + AC$ or $DB + BC$.



MA335.

a) Find all geometric series such that the sum of the first two terms is 2 and the sum of the first three terms is 3.

b) For each of the sequences determined in part a), calculate the sum of all terms having value less than 1.

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Les problèmes proposés dans cette section sont appropriés aux étudiants de l'école secondaire.

Cliquez ici afin de soumettre vos solutions, commentaires ou généralisations aux problèmes proposés dans cette section.

Pour faciliter l'examen des solutions, nous demandons aux lecteurs de les faire parvenir au plus tard le **15 novembre 2025**.

MA331. *Soumis par Ivan Hadinata.*

Soit M le nombre de paires ordonnées de nombres naturels (a, b) satisfaisant l'équation

$$a^b = (20!)^{24!}.$$

Trouvez les trois derniers chiffres de M.

MA332. *Soumis par Michael Friday.*

Soit ABC un triangle quelconque et soit H et O respectivement l'orthocentre et le centre du cercle circonscrit au triangle. Soit encore M_a, M_b et M_c respectivement les milieux des côtés BC, CA et AB. Montrez que

$$OH^2 = (HM_a^2 - OM_a^2) + (HM_b^2 - OM_b^2) + (HM_c^2 - OM_c^2)$$