

Sparse Harmonic series

Let $S = \{n_1, n_2, \dots\}$ denote the set of positive integers which do not contain the digit 0 in their decimal representation. Show that $\sum_{k=1}^{\infty} \frac{1}{n_k}$ converges.

Note: this gives the surprising result that removing certain values from the divergent harmonic series ends up giving a convergent series.