Separation of Variables - division by 0

Normally when we use separation of variables to solve

$$\frac{dy}{dx} = g(x)h(y)$$

we divide by $h(y)$ and ignore the fact that this function may be 0. In actuality, we should be checking the cases where $h(y) = 0$ and $h(y) \neq 0$ separately, for e.g.

$$\begin{aligned}
\frac{dy}{dx} &= 2xy && \\
\frac{1}{y}\,dy &= 2x\,dx && \text{if } y \neq 0 \\
\int \frac{1}{y}\,dy &= \int 2x\,dx && \text{if } y \neq 0 \\
\ln|y| &= x^2 + C_1 && \text{where } C_1 \text{ is a constant} \\
|y| &= e^{x^2+C_1} = C_2 e^{x^2} && \text{where } C_2 \text{ is a positive constant} \\
y &= C_3 e^{x^2} && \text{where } C_3 \text{ is a non-zero constant}
\end{aligned}$$

In the case of $y = 0$, notice that this is indeed a solution to the given differential equation, so our solution can be written as

$$\begin{aligned}
y &= C_3 e^{x^2} && \text{where } C_3 \text{ is a non-zero constant, if } y \neq 0, \\
\text{or } y &= 0
\end{aligned}$$

However, notice that if we replace C_3 with 0, then the solution $y = 0$ is recovered, so we may instead write (the more usual presentation of) the solution as:

$$y = Ce^{x^2} \qquad \text{where } C \text{ is a constant}$$

That means that if we completely ignored the fact that we can't divide by 0 (or forget), we get the right answer anyway!

Can you come up with a differential equation in which the division by 0 causes an issue? That is, if you don't consider the case where $h(y) = 0$ separately, you get the wrong answer.