

Differentiability for piecewise functions

Suppose

$$f(x) = \begin{cases} g(x) & \text{if } x < a \\ k & \text{if } x = a \\ h(x) & \text{if } x > a \end{cases}$$

We know that $f(x)$ is differentiable at $x = a$ if

$$\lim_{x \rightarrow a^-} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow a^+} \frac{f(x) - f(a)}{x - a}$$

In this question, we find another criteria for differentiability. Prove that if $f(x)$ is continuous at a and if $g'(a) = h'(a)$, then $f(x)$ is differentiable at $x = a$.