A nowhere continuous function

(Hard) Does there exist a function $f : \mathbb{R} \to \mathbb{R}$ (i.e., the domain is the real numbers and every value is a real number) such that $f(x)$ is not continuous at any $a \in \mathbb{R}$?