

## 2024–2025 Winter Break Math Challenges

### Challenge 1: Digit puzzle

To ring in the new year, make the number 2025 in 9 different ways, each time by using copies of the same digit and the following operations (in addition to parentheses):

- Standard operations:  $+$ ,  $-$ ,  $\times$ ,  $\div$
- Negation:  $-\square$
- Exponentiation of two numbers:  $\square^\square$
- Square root of a number:  $\sqrt{\square}$
- Factorial:  $\square!$  (Note: you may use iterated factorial but not multi-factorial, so that  $3!! = (3!)! = 6! = 720$ , and **not**  $3!! = 3 \times 1 = 3$ .)
- Concatenation (i.e. “glueing”) of digits (only of the original digit used):  $dd$

Your score for a particular digit is the number of copies you use, and your goal is to have the lowest score possible.

For example, you can make 2025 by using copies of the digit “9” as follows:

$$2025 = \underbrace{\frac{9}{9} + \frac{9}{9} + \cdots + \frac{9}{9}}_{2025 \text{ times}}$$

If you do it like this, you are using 4050 copies of 9, which is not good for you. A far more efficient way to do it is

$$2025 = 99 \times 9 \times 9 - 9 \times 9 \times 9 \times 9 + 99 \times 9 - 99 \times \sqrt{9} - 9 \times \sqrt{9}$$

which gets you there with only 16 copies (this is, of course, not optimal).

- (1) Using the digit 1
- (2) Using the digit 2
- (3) Using the digit 3
- (4) Using the digit 4
- (5) Using the digit 5
- (6) Using the digit 6
- (7) Using the digit 7
- (8) Using the digit 8
- (9) Using the digit 9

## Challenge 2: Approximations

To say goodbye to the last year, here is a puzzle about 2024. How close can you get to 2024 using the digits 2, 0, 2, 4 exactly once? You may use the following operations (in addition to parentheses):

- Standard operations:  $+$ ,  $-$ ,  $\times$ ,  $\div$
- Negation:  $-\square$
- Exponentiation of two numbers:  $\square^\square$
- Square root of a number:  $\sqrt{\square}$
- Factorial:  $\square!$ 
  - Note: you may use iterated factorial but not multi-factorial, so that  $3!! = (3!)! = 6! = 720$ , and **not**  $3!! = 3 \times 1 = 3$ .
  - Hint:  $0! = 1$
- Floor:  $\lfloor \square \rfloor$  (this is the largest integer less than or equal to the input, so  $\lfloor \pi \rfloor = 3$ )

Notice you CANNOT use "glueing"!

### Challenge 3: Pixel art

Make a pixel art by colouring cells in a grid. It must be

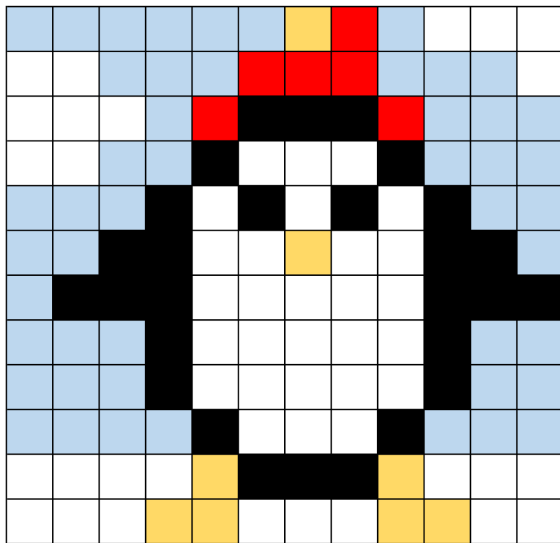
- (1) winter-themed, and
- (2) math-related.

You must also write a short paragraph explaining how your art satisfies these two points. You will be judged on:

- (1) the aesthetic and technical quality of the art,
- (2) the depth of connection to mathematics, and
- (3) the quality of writing.

Your art piece must consist of multiple colours. For each colour, indicate the fraction of the picture which has that colour, both as a fraction out of the size of the grid and as a reduced, simplified fraction.

For example, consider the following which was made on a  $12 \times 12$  grid:



Colour	Fraction	Reduced fraction
Black	$\frac{28}{144}$	$\frac{7}{36}$
White	$\frac{54}{144}$	$\frac{3}{8}$
Blue	$\frac{48}{144}$	$\frac{1}{3}$
Red	$\frac{6}{144}$	$\frac{1}{24}$
Yellow	$\frac{8}{144}$	$\frac{1}{18}$

#### Challenge 4: Poem

“Pilish” is writing text in which the lengths of successive words represents the digits of  $\pi$ . Michael Kieth once rewrote Edgar Allen Poe’s poem ”The Raven” to do just that. While the full poem has 18 stanzas, I present here only the title and first stanza as an example:

Poe, E.  
Near a Raven

Midnights so dreary, tired and weary,  
Silently pondering volumes extolling all by-now obsolete lore,  
During my rather long nap — the weirdest tap!  
An ominous vibrating sound disturbing my chamber’s antedoor.  
“This,” I whispered quietly, “I ignore.”

Notice the number of letters in each word: 3.14159265358979323846264338327950288419716

To get the digit “0,” a 10-letter word is used (further details and rules about how to count characters for digits can be found at <http://www.cadaeic.net/pilish.htm>).

Here is the challenge: compose such a poem about winter that uses between 10 and 50 digits of  $\pi$ . You will be judged on:

- (1) form,
- (2) rhyme or rhythm,
- (3) poetic devices,
- (4) coherence,
- (5) imagery or mood, and
- (6) significance.