2023–2024 Winter Break Math Challenges

Challenge 1: Digit puzzle

To ring in the new year, make the number 2024 in 9 different ways, each time by using copies of the same digit and the following operations (in addition to parentheses):

- Standard operations: $+, -, \times, \div$
- Negation: $-\square$
- Exponentiation of two numbers: $\square^{\square}$
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$ (Note: you may use iterated factorial but not muilti-factorial, so that $3!! = (3!)! = 6! = 720$, and **not** $3!! = 3 \times 1 = 3$.)
- Concatenation (i.e. "glueing") of digits (only of the original digit used): dd

Your score for a particular digit is the number of copies you use, and your goal is to have the lowest score possible.

For example, you can make 2024 by using copies of the digit "9" as follows:

$$2024 = \underbrace{\frac{9}{9} + \frac{9}{9} + \cdots + \frac{9}{9}}_{\text{2024 times}}.$$

If you do it like this, you are using 4048 copies of 9, which is not good for you. A far more efficient way to do it is

$$2024 = 99 \times 9 \times 9 - 9 \times 9 \times 9 \times 9 + 99 \times 9 - 99 \times \sqrt{9} - 9 \times \sqrt{9} - \frac{9}{9}$$

which gets you there with only 18 copies (this is, of course, not optimal).

(1) Using the digit 1

(2) Using the digit 2

(3) Using the digit 3

(4) Using the digit 4

(5) Using the digit 5

(6) Using the digit 6

(7) Using the digit 7

(8) Using the digit 8

(9) Using the digit 9

Challenge 2: Approximations

How close can you get to 2024 using the digits $2, 0, 2, 4$ exactly once? You may use the following operations (in addition to parentheses):

- Standard operations: $+, -, \times, \div$
- Negation: $-\square$
- Exponentiation of two numbers: $\square^{\square}$
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$
  - Note: you may use iterated factorial but not multi-factorial, so that $3!! = (3!)! = 6! = 720$, and **not** $3!! = 3 \times 1 = 3$.
  - Hint: $0! = 1$
- Floor: $\lfloor \square \rfloor$ (this is the largest integer less than or equal to the input, so $\lfloor \pi \rfloor = 3$)

Notice you CANNOT use "glueing"!

Challenge 3: Sequences

Find a good rule for describing the following sequences. If possible, also write down an algebraic formula to give the nth term.

(a) $1, 1, 2, 2, 3, 4, 4, 8, 5, 16, 6, \ldots$

(b) $0, 1, 1, 0, 0, 2, 4, 2, 1, 4, 12, 11, \frac{11}{3}, \ldots$

(c) $2, 5, 7, 3, 13, 3, 19, 3, 5, 31, 3, 3, 43, 3, 7, 5, 61, 3, 3, 73, 3, 3, 5, 7, 3, 103, \ldots$

(d) $2, 3, 22, 5, 23, 7, 222, 33, 25, 11, 223, 13, 27, 35, 2222, 17, 233, 19, 225, 37, 112, 23, 2223, 55, 123, \ldots$

(e) $2, 7, 15, 5, 16, 33, 11, 34, 69, 23, 70, 141, 47, 142, 285, 95, 286, 573, 191, 574, 1149, 383, 1150, 2301, , \ldots$

Challenge 4: Geometric construction

Create a winter-themed picture, by hand, using only a pencil/pen, ruler, compass, and protractor. Points will be given based on creativity, aesthetics, and mathematical accuracy.