Series Convergence & Divergence Quick Reference Sheet

1. Convergence and Divergence Tests

Test	When to Use	Conditions	Conclusions
Divergence	First resort for any series.	$\lim_{n\to\infty} a_n \neq 0$	$\sum a_n$ diverges. NOTE: if the limit is 0, test is inconclusive.
Integral	Terms look like a function you can easily integrate.	$f(n) = a_n$. $f(x)$ positive, decreasing, and continuous.	$\int_1^\infty f(x)dx$ and $\sum a_n$ either both converge or both diverge.
Comparison	When a_n looks like a p-series or geometric series.	$0 \leq a_n \leq b_n$	$\sum b_n$ converges $\Rightarrow \sum a_n$ converges. $\sum a_n$ diverges $\Rightarrow \sum b_n$ diverges.
Limit Comp.	When a_n looks like a p-series or geometric series (LCT more than DCT).	$a_n, b_n > 0$; $L = \lim_{n\to\infty} \frac{a_n}{b_n}$	**1.** $0 < L < \infty$: both conv. or div. **2.** $L = 0, \sum b_n$ conv: $\sum a_n$ conv. **3.** $L = \infty, \sum b_n$ div: $\sum a_n$ div.
Alternating	Series is alternating in sign.	$\sum(-1)^n b_n$ $(b_n > 0)$, $b_n \searrow 0$	$\sum a_n$ converges.
Ratio	High-priority for factorials ($n!$) or n in the exponent.	$L = \lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert$	Converges if $L < 1$ Diverges if $L > 1$ Inconclusive if $L = 1$.
Geometric	Given constants raised to the n.	$\sum_{n=1}^{\infty} ar^{n-1}$	Converges to $\frac{a}{1-r}$ if $\lvert r\rvert < 1$. Diverges if $\lvert r\rvert \geq 1$.
p-Series	Of the form $1/n^p$.	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$. Diverges if $p \leq 1$.

2. Convergence Definitions

- **Partial Sum:** $S_N = \sum_{n=1}^{N} a_n$. $\sum a_n$ *converges* (to the limit) if $\lim_{N\to\infty} S_N$ exists, otherwise it *diverges*. (Can also be a convergence/divergence test.)
- **Absolute Convergence:** A series $\sum a_n$ *converges absolutely* if the series of absolute values $\sum \lvert a_n\rvert$ converges. (Also implies convergence.)
- **Conditional Convergence:** A series $\sum a_n$ *converges conditionally* if it converges but $\sum \lvert a_n\rvert$ diverges.