8.3 example SOLUTIONS

1. The volume of water in an above-ground pool is 45 000 L. A pump can move water at a rate of

$$r_{\text{pump}}(t) = \frac{20\left(35 + \log_{10}\left(\frac{t+1}{100}\right)\right)}{\frac{t}{100} + 3} \text{ L/min}$$

where t is time measured in minutes.

(a) How much water would be in the pool if water is pumped in for 30 minutes?

Solution:

$$45000 + \int_0^{30} r_{\text{pump}}(t)\,dt \approx 51499.8 \text{ L}$$

(b) How much water would be in the pool if water is pumped out for 30 minutes?

Solution:

$$45000 - \int_0^{30} r_{\text{pump}}(t)\,dt \approx 38500.2 \text{ L}$$

(c) Suppose the pump is being used to move water into the pool. Suppose further that there is a leak in the pool, where water is leaking at a rate of

$$r_{\text{leak}}(t) = 225 - t\sin\frac{t}{11}$$

On the interval $t \in [0, 30]$:

(i) When is the pool at its fullest?

(ii) When is the pool at its emptiest?

(iii) Is the volume of the pool increasing or decreasing at $t = 30$?

Solution: We have $V(t) = 45000 + \int_0^t r_{\text{pump}}(x)\,dx - \int_0^t r_{\text{leak}}(x)\,dx$. Then $V'(t) = r_{\text{pump}}(t) - r_{\text{leak}}(t)$, which has critical points $t \approx 6.9635, 28.3987$. We apply the candidates test on the critical points and endpoints:

t	$V(t)$
0	45000
6.9635	44984.794
28.3987	45103.602
30	44899.415

(i) The pool at its fullest at ≈ 6.964 minutes.

(ii) The pool at its emptiest at ≈ 28.399 minutes.

(iii) $V'(30) = r_{\text{pump}}(30) - r_{\text{leak}}(30) < 0$, so the volume is decreasing at 30 minutes.