8.2 example SOLUTIONS

1. On $t \in [0,3]$ in seconds, we have two particles described by:

Particle	position (m)	velocity (m/s)	acceleration (m/s^2)
A	$s_A = \dfrac{t(1-t)^2}{t+1} + 2$	$v_A = \dfrac{2t^3 + t^2 - 4t + 1}{(t+1)^2}$	$a_A = \dfrac{2(t^3 + 3t^2 + 3t - 3)}{(t+1)^3}$
B	$s_B = t\ln(t+1)$	$v_B = \dfrac{t}{t+1} + \ln(t+1)$	$a_B = \dfrac{t+2}{(t+1)^2}$

(a) Calculate $\dfrac{1}{3}\displaystyle\int_0^3 v_A \, dt$ and interpret the result, including units.

Solution: 1 m/s. This is the average velocity of particle A over $[0,3]$.

(b) When is particle A slowing down?

Solution: We need $v_A a_A < 0$.
For $v_A > 0, a_A < 0$, we get $0 < t < 0.281$.
For $v_A < 0, a_A > 0$, we get $0.587 < t < 1$.

(c) Is the distance between particles A and B increasing or decreasing at:
(i) $t = 2$?
(ii) $t = 3$?

Solution: Let $s(t) = s_A(t) - s_B(t)$ and $v(t) = v_A(t) - v_B(t)$. Notice that distance increasing means $sv > 0$. In this case, we can check that $s(t) > 0$, so we only need to see when $v(t) > 0$ to determine when the distance is increasing.

(i) At $t = 2$, $v(2) < 0$, so the distance between particles A and B is decreasing.

(ii) At $t = 3$, $v(3) > 0$, so the distance between particles A and B is increasing.

(d) What are the maximum and minimum distances between particles A and B?

Solution: We can check that $s_A > s_B$ on this interval, so it suffices to check the extrema of $s(t)$. To do so, we check critical points of $v(t)$. $v(t)$ is defined on all of $[0,3]$, and $v(t) = 0$ at $t \approx 0.16133, 2.24433$. We apply the candidates test on the critical points and endpoints:

t	$d(t)$
0	2
0.16133	2.073
2.24433	0.430
3	0.841

The maximum distance is 2.073 m and the minimum distance is 0.430 m.