7.9 Logistic model formulae derivation

Form 1:

$$\begin{aligned}
\frac{dy}{dt} &= Ky(L-y) \\
\int \frac{1}{y(L-y)}\,dy &= \int K\,dx \\
\int \left(\frac{1/L}{y} + \frac{1/L}{L-y}\right) dy &= Kx + C_1 \\
\frac{1}{L}\left(\ln|y| - \ln|L-y|\right) &= Kx + C_1 \\
\ln\left|\frac{y}{L-y}\right| &= KLx + C_2 \\
\frac{y}{L-y} &= e^{KLx+C_2} = e^{C_2}e^{KLx} = C_3e^{KLx} \\
y &= (L-y)C_3e^{KLx} = C_3Le^{KLx} - C_3e^{KLx}y \\
(1+C_3e^{KLx})y = y + C_3e^{KLx}y &= C_3Le^{KLx} \\
y &= \frac{C_3Le^{KLx}}{1+C_3e^{KLx}} \\
y &= \frac{L}{C_4e^{-KLx}+1} = \frac{L}{1+Ce^{-KLx}}
\end{aligned}$$

Note: I have used different constants and denoted them with differing subscripts as an aid in learning - it is not necessary to distinguish the different constants on an FRQ. I have also shown more steps than are necessary.

Form 2:

$$\begin{aligned}
\frac{dy}{dt} &= ky\left(1-\frac{y}{L}\right) \\
\int \frac{1}{y(1-\frac{y}{L})}\,dy &= \int k\,dx \\
\int \left(\frac{1}{y}+\frac{1/L}{1-\frac{y}{L}}\right)\,dy &= kx + C_1 \\
\ln|y| - \ln\left|1-\frac{y}{L}\right| &= kx + C_1 \\
\ln\left|\frac{y}{L-y}\right| &= kx + C_2 \\
\frac{y}{L-y} &= e^{kx+C_2} = e^{C_2}e^{kx} = C_3e^{kx} \\
y &= (L-y)C_3e^{kLx} = C_3Le^{kx} - C_3e^{kx}y \\
(1+C_3e^{kx})y = y + C_3e^{kx}y &= C_3Le^{kx} \\
y &= \frac{C_3Le^{kx}}{1+C_3e^{kx}} \\
y &= \frac{L}{C_4e^{-kx}+1} = \frac{L}{1+Ce^{-kx}}
\end{aligned}$$

Note: I have used different constants and denoted them with differing subscripts as an aid in learning - it is not necessary to distinguish the different constants on an FRQ. I have also shown more steps than are necessary.