6.14 Practice

For each integral, determine what technique would be useful to solve them. (For additional practice, solve the integral.)

1. $\displaystyle\int \frac{a^3-1}{a^2+1}\,da$

> **Solution:** Long division
>
> Answer:
> $$\frac{1}{2}a^2 - \frac{1}{2}\ln(a^2+1) - \arctan a + C$$

2. $\displaystyle\int \frac{1}{x^2+2x+2}\,dx$

> **Solution:** Complete the square
>
> Answer:
> $$\arctan(x+1) + C$$

3. $\displaystyle\int \frac{x}{x^2+2x+2}\,dx$

> **Solution:** Complete the square, split the integral
>
> Answer:
> $$\frac{1}{2}\ln|x^2+2x+2| - \arctan(x+1) + C$$

4. $\displaystyle\int\int \frac{x+1}{x^2+2x+2}\,dx$

> **Solution:** Logarithmic integration (substitute $u = x^2+2x+2$)
>
> Answer:
> $$\frac{1}{2}\ln|x^2+2x+2| + C$$

5. $\displaystyle\int \frac{x-1}{x^2+2x+3}\,dx$

> **Solution:** Complete the square, split the integral, scale to get an arctan integral
>
> Answer:
> $$\frac{1}{2}\ln|x^2+2x+3| - \sqrt{2}\arctan\left(\frac{x+1}{\sqrt{2}}\right) + C$$

6. $\displaystyle\int_0^1 t(1-t)^{10}\,dt$

Solution: Substitute $u = t - 1$

Answer:

$$\frac{1}{132}$$

7. $\displaystyle\int x(x-1)(x-2)\,dx$

Solution: Expand

Answer:

$$\frac{1}{4}x^4 - x^3 + x^2 + C$$

8. $\displaystyle\int_1^3 r\sqrt{r^2-1}\,dr$

Solution: Substitute $u = r^2 - 1$

Answer:

$$\frac{16\sqrt{2}}{3}$$

9. $\displaystyle\int_e^{e^2} \frac{1}{x\ln x}\,dx$

Solution: Substitute $u = \ln x$

Answer:

$$\ln 2$$

10. $\displaystyle\int_0^{\pi/6} \frac{\cos\theta - \cos^3\theta}{\sin^2\theta}\,d\theta$

Solution: Algebraic manipulation: trig identity and simplifying the fraction

Answer:

$$\frac{1}{2}$$

11. $\displaystyle\int_{-2}^{2} x^3 \sin(x^2+1)\,dx$

> **Solution:** Odd function over a symmetric interval, or use substitution $u = x^2 + 1$
>
> Answer:
> $$0$$

12. $\displaystyle\int \frac{1}{\sqrt{u}e^{\sqrt{u}}}\,du$

> **Solution:** Substitute $x = \sqrt{u}$
>
> Answer:
> $$-2e^{-\sqrt{u}} + C$$

13. $\displaystyle\int \frac{1}{\sqrt{1-x-x^2}}\,dx$

> **Solution:** Complete the square
>
> Answer:
> $$\arcsin\left(\frac{2x+1}{\sqrt{5}}\right) + C$$

14. $\displaystyle\int \frac{2^{\sin\theta}}{\sec\theta}\,d\theta$

> **Solution:** Substitute $u = \sin\theta$ and use $\frac{1}{\sec\theta} = \cos\theta$
>
> Answer:
> $$\frac{2^{\sin\theta}}{\ln 2} + C$$

15. $\displaystyle\int_{-2}^{2} (x + x^2 + x^7 + \sin x)\,dx$

> **Solution:** Most of this is an odd function over a symmetric interval, then an easy integral of x^2
>
> Answer:
> $$\frac{16}{3}$$