Mini-math Div 3/4: Monday, November 2, 2026 (6.9-6.14) - 12 minutes

SOLUTIONS

1. (1 point) Using the substitution $u = x^3 - 2$, $\displaystyle\int_{-2}^{3} x^2(x^3-2)^3\,dx$ is equal to which of the following?

   A. $3\displaystyle\int_{-10}^{25} u^3\,du$ B. $\displaystyle\int_{-10}^{25} u^3\,du$ C. $\dfrac{1}{3}\displaystyle\int_{-10}^{25} u^3\,du$ D. $\displaystyle\int_{-2}^{3} u^3\,du$ E. $\dfrac{1}{3}\displaystyle\int_{-2}^{3} u^3\,du$

   Solution: Changing the bounds: $-2 \mapsto (-2)^3 - 2 = -8 - 2 = -10$ and $3 \mapsto 3^3 - 2 = 27 - 2 = 25$. Since $u = x^3 - 2$, we have $du = 3x^2\,dx$, so $x^2\,dx = \frac{1}{3}\,du$. Thus (C) is correct.

2. (1 point) $\displaystyle\int_0^1 \frac{2x-3}{x^2-5x+6}\,dx$ is

   A. $\ln 8$ B. $\ln\left(\dfrac{27}{2}\right)$ C. $\ln 432$ D. $\ln\left(\dfrac{16}{27}\right)$ E. $\ln\left(\dfrac{27}{16}\right)$

   Solution:

   $$\int_0^1 \frac{2x-3}{(x-2)(x-3)}\,dx = \int_0^1 \left(\frac{-1}{x-2} + \frac{3}{x-3}\right)\,dx = \Big(-\ln|x-2| + 3\ln|x-3|\Big)\Big|_0^1 = 4\ln 2 - 3\ln 3 = \ln\frac{16}{27}.$$

   (D) is correct.

3. (1 point) $\displaystyle\int_1^\infty xe^{-x^2}\,dx$ is

   A. $-\dfrac{1}{e}$ B. $\dfrac{1}{2e}$ C. $\dfrac{1}{e}$ D. $\dfrac{2}{e}$ E. divergent

   Solution:

   $$\int_1^\infty xe^{-x^2}\,dx = \lim_{b\to\infty}\int_1^b xe^{-x^2}\,dx = \lim_{b\to\infty} -\frac{1}{2}e^{-x^2}\Big|_1^b = \lim_{b\to\infty} -\frac{1}{2}\big(e^{-b^2} - e^{-1}\big) = \frac{1}{2e}.$$

   (B) is correct.

4. (1 point) Find $\displaystyle\int \frac{dx}{\sqrt{4x-x^2}}$

(A) $\arcsin(x-2)+C$ (B) $\dfrac{1}{2}\arcsin\left(\dfrac{x-2}{2}\right)+C$ (C) $2\arcsin(x-2)+C$

(D) $2\arcsin\left(\dfrac{x-2}{2}\right)+C$ (E) $\arcsin\left(\dfrac{x-2}{2}\right)+C$

Solution: omplete the square and factor out the 4 under the root:

$$4x-x^2 = 4-(x-2)^2 = 4\left(1-\left(\tfrac{x-2}{2}\right)^2\right),$$

so

$$\int \frac{dx}{\sqrt{4x-x^2}} = \int \frac{dx}{2\sqrt{1-\left(\tfrac{x-2}{2}\right)^2}}.$$

Since $\frac{d}{dx}\left(\frac{x-2}{2}\right) = \frac{1}{2}$, the factor of $\frac{1}{2}$ in front is exactly what is needed, and the integral is $\arcsin\left(\frac{x-2}{2}\right)+C$. (E) is correct.

5. (1 point) Find $\displaystyle\int (3x-1)\sin x\,dx$

(A) $(1-3x)\cos x+3\sin x+C$ (B) $(3x-1)\cos x+3\sin x+C$ (C) $-(3x-1)\cos x-3\sin x+C$

(D) $(3x-1)\sin x+\cos x+C$ (E) $(1-3x)\sin x-3\cos x+C$

Solution: Let $u=3x-1$ and $dv=\sin x\,dx$, so $du=3\,dx$ and $v=-\cos x$. Then

$$\int (3x-1)\sin x\,dx = uv-\int v\,du = -(3x-1)\cos x+3\int \cos x\,dx = (1-3x)\cos x+3\sin x+C.$$

(A) is correct.

6. (1 point) Which technique is most appropriate for evaluating $\displaystyle\int x\ln x\,dx$?

(A) substitution $u=\ln x$ (B) partial fractions (C) integration by parts

(D) completing the square (E) long division

Solution: The integrand is a product of an algebraic function and a logarithm, and no inner function has its derivative appearing as a factor, so substitution does not simplify it. The integrand is not rational and has no quadratic under a root, so the other techniques do not apply. Integration by parts (with $u=\ln x$) is the appropriate technique. (C) is correct.