

SOLUTIONS

1. (1 point) Water flows into a tank at a rate of  $r(t)$  liters per minute, where  $t$  is in minutes. Which expression represents the total amount of water that flows into the tank from  $t = 2$  to  $t = 5$  minutes?

A.  $r(5) - r(2)$     B.  $r'(5) - r'(2)$     C.  $\int_2^5 r(t) \, dt$     D.  $\int_2^5 r'(t) \, dt$     E.  $r(5) \cdot (5 - 2)$

**Solution:** The definite integral of a rate of change gives the net accumulation: total water =  $\int_2^5 r(t) \, dt$ , so (C) is correct.

2. (1 point) Suppose  $V(x) = \int_0^{x^2} \sin t \, dt$ . What is the derivative,  $V'(x)$ ?

A.  $\cos x$     B.  $\sin x$     C.  $\sin x^2$     D.  $2x \sin x^2$     E.  $2x \cos x^2$

**Solution:** By FTC I and the chain rule,  $V'(x) = \sin(x^2) \cdot \frac{d}{dx}(x^2) = 2x \sin(x^2)$ , so (D) is correct.

3. (1 point)  $\int_1^8 t^{-2/3} \, dt =$

A.  $-3$     B.  $-1$     C.  $\frac{93}{160}$     D.  $1$     E.  $3$

**Solution:**

$$\int_1^8 t^{-2/3} \, dt = 3t^{1/3} \Big|_1^8 = 3(2 - 1) = 3.$$

(E) is correct.

4. (1 point) Given  $\int_1^7 f(x) dx = 4$ ,  $\int_{-1}^7 f(x) dx = -3$ , and  $\int_1^5 f(x) dx = 6$ , find  $\int_{-1}^5 (2f(x) + 3) dx$
- A. -2                      B. 15                      C. 16                      D. 17                      E. 28

**Solution:**

$$\int_{-1}^5 f(x) dx = \int_{-1}^7 f(x) dx + \int_7^1 f(x) dx + \int_1^5 f(x) dx = -3 - 4 + 6 = -1$$

Therefore

$$\int_{-1}^5 (2f(x) + 3) dx = 2(-1) + \int_{-1}^5 3 dx = -2 + 3(6) = 16.$$

(C) is correct.

5. (1 point) Evaluate  $\int_0^{\pi/2} (2 \cos x - \sin x) dx$ .
- A. 1                      B. 2                      C. 3                      D.  $\pi - 1$                       E.  $\pi + 1$

**Solution:** By FTC II,

$$\int_0^{\pi/2} (2 \cos x - \sin x) dx = [2 \sin x + \cos x]_0^{\pi/2} = (2 \cdot 1 + 0) - (0 + 1) = 1.$$

(A) is correct.

6. (1 point) Choose the integral that is the limit of the Riemann Sum:  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \sin\left(1 + \frac{8k}{n}\right) \cdot \frac{4}{n}$
- A.  $\int_0^4 \sin(1 + 2x) dx$     B.  $\int_1^5 \sin(1 + x) dx$     C.  $\int_1^5 \sin(1 + 2x) dx$     D.  $\int_1^5 \sin x dx$

**Solution:** Write

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \sin\left(1 + 2 \cdot \frac{4k}{n}\right) \cdot \frac{4}{n}.$$

This is a right Riemann sum with  $\Delta x = \frac{4}{n}$ ,  $a = 0$ ,  $b = 4$ , and  $x_k = \frac{4k}{n}$ , so the integrand is  $\sin(1 + 2x)$ .

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \sin\left(1 + \frac{8k}{n}\right) \cdot \frac{4}{n} = \int_0^4 \sin(1 + 2x) dx.$$

(A) is correct.

7. (1 point) Suppose  $f$  is concave up and the following are selected values of  $f$ :

|        |   |   |   |   |    |
|--------|---|---|---|---|----|
| $x$    | 0 | 1 | 3 | 4 | 6  |
| $f(x)$ | 3 | 2 | 4 | 6 | 12 |

Using the trapezoidal rule with 4 unequal subintervals to approximate  $\int_0^6 f(x) dx$ , then:

- A.  $\int_0^6 f(x) dx \approx 31.5$  and this is an underestimate
- B.  $\int_0^6 f(x) dx \approx 31.5$  and this is an overestimate
- C.  $\int_0^6 f(x) dx \approx 63$  and this is an underestimate
- D.  $\int_0^6 f(x) dx \approx 63$  and this is an overestimate

**Solution:**

$$\frac{3+2}{2} \cdot 1 + \frac{2+4}{2} \cdot 2 + \frac{4+6}{2} \cdot 1 + \frac{6+12}{2} \cdot 2 = 2.5 + 6 + 5 + 18 = 31.5.$$

Since  $f$  is concave up, each trapezoid lies above the curve, so the trapezoidal rule is an overestimate, so (B) is correct.

8. (1 point) Choose the limit of the Riemann Sum that is the integral:  $\int_2^4 \frac{1}{x+2} dx$

A.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\frac{k}{n} + 2} \cdot \left(\frac{2}{n}\right)$

C.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\frac{k}{n} + 4} \cdot \left(\frac{2}{n}\right)$

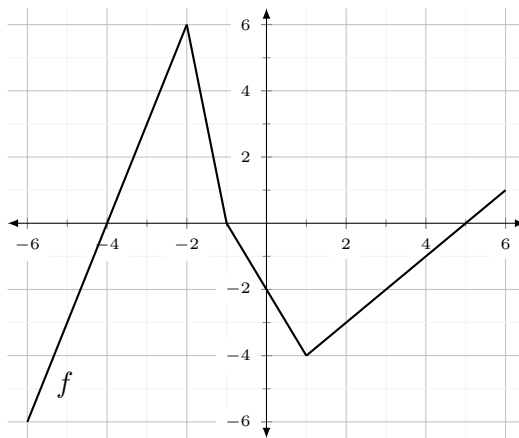
B.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\frac{2k}{n} + 2} \cdot \left(\frac{2}{n}\right)$

D.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\frac{2k}{n} + 4} \cdot \left(\frac{2}{n}\right)$

**Solution:**  $\Delta x = \frac{4-2}{n} = \frac{2}{n}$  and  $x_k = a + \Delta x = 2 + \frac{2k}{n}$ . With  $f(x) = \frac{1}{x+2}$ ,  $f(x_k) = \frac{1}{2 + \frac{2k}{n} + 2} = \frac{1}{\frac{2k}{n} + 4}$ , so (D) is correct.

9. (1 point) To the right is a graph of  $f(x)$ . Suppose  $g(x) = \int_a^x f(t) dt$  and  $g(1) = 3$ . What is the minimum value of  $g(x)$  on  $[-6, 2]$ ?

- A. -8
- B. -5
- C. -4
- D. -3
- E. -2



**Solution:**  $g$  has a local minimum where  $g' = f$  changes from negative to positive (at  $x = -4$ ), or at an endpoint. By the Net Change Theorem,

$$g(b) = g(1) + \int_1^b f(t) dt = 3 + \int_1^b f(t) dt.$$

Computing the signed areas from the graph:

$$g(-6) = 3 + \int_1^{-6} f = 3 + [-(-6 + 6 + 3 - 4)] = 3 + 1 = 4,$$

$$g(-4) = 3 + \int_1^{-4} f = 3 - (6 + 3 - 4) = 3 - 5 = -2,$$

$$g(2) = 3 + \int_1^2 f = 3 + \frac{1}{2}(1)(-4 + -3) = 3 - 3.5 = -0.5.$$

The minimum is -2 at  $x = -4$ . (E) is correct.