Renert School: Integration Bee 2026–2027

Problem 1.

$$\int_1^{mower} \frac{1}{u}\,du$$

Solution:

$$\int_1^{mower} \frac{1}{u}\,du = \ln|u|\Big|_1^{mower} = \ln|mower|$$

Problems 2-17 are great review.

Problem 2.

$$\int_{-8}^{-1} \left(\frac{2}{x^{1/3}} + 4x\right)\,dx$$

Solution:

$$\begin{aligned}
&\int_{-8}^{-1} \left(\frac{2}{x^{1/3}} + 4x\right)\,dx \\
&= \left(3x^{2/3} + 2x^2\right)\Big|_{-8}^{-1} \\
&= 3((-1)^{2/3} - (-8)^{2/3}) + 2((-1)^2 - (-8)^2) \\
&= 3(1-4) + 2(1-64) \\
&= -9 - 126 = -135
\end{aligned}$$

Problem 3.

$$\int \frac{\sqrt{r}-5}{\sqrt{r}}\,dr$$

Solution: Divide to get

$$\int \frac{\sqrt{r}-5}{\sqrt{r}}\,dr = \int (1 - 5r^{-1/2})\,dr = r - 10\sqrt{r} + C$$

Problem 4.

$$\int s(\sqrt{s}+1)\,ds$$

Solution: Expand to get

$$\int s(\sqrt{s}+1)\,ds = \int (s^{3/2} + s)\,ds = \frac{2}{5}s^{5/2} + \frac{1}{2}s^2 + C$$

Problem 5.

$$\int_{\pi/6}^{\pi/3} \sin(2x)\,dx$$

Solution:

Method 1: Using $u = 2x$, $du = 2\,dx$, $\pi/6 \mapsto \pi/3$, $\pi/3 \mapsto 2\pi/3$,

$$\begin{aligned}\int_{\pi/6}^{\pi/3} \sin(2x)\,dx &= \int_{\pi/3}^{2\pi/3} \frac{1}{2}\sin u\,du \\ &= -\frac{1}{2}\cos u\Big|_{\pi/3}^{2\pi/3} = -\frac{1}{2}\left(-\frac{1}{2} - \frac{1}{2}\right) = \frac{1}{2}\end{aligned}$$

Method 2: Integrating directly,

$$\int_{\pi/6}^{\pi/3} \sin(2x)\,dx = -\frac{1}{2}\cos(2x)\Big|_{\pi/6}^{\pi/3} = -\frac{1}{2}\left(-\frac{1}{2} - \frac{1}{2}\right) = \frac{1}{2}$$

Problem 6.

$$\int x^3 e^{x^4}\,dx$$

Solution: Using $u = x^4$,

$$\int x^3 e^{x^4}\,dx = \int \frac{1}{4}e^u\,du = \frac{1}{4}e^{x^4} + C$$

Problem 7.

$$\int (2x-1)\sin(4x)\,dx$$

Solution: Use $f = 2x - 1$, $g' = \sin(4x)$, so $f' = 2$, $g = -\frac{1}{4}\cos(4x)$. By IBP,

$$\begin{aligned}\int (2x-1)\sin(4x)\,dx &= -\frac{2x-1}{4}\cos(4x) + \frac{1}{2}\int \cos(4x)\,dx \\ &= -\frac{2x-1}{4}\cos(4x) + \frac{1}{8}\sin(4x) + C\end{aligned}$$

Problem 8.

$$\int \frac{\sec\theta\tan\theta}{\sqrt{\sec\theta + 1}}\,d\theta$$

Solution: Using $u = \sec\theta + 1$, $du = \sec\theta\tan\theta\, d\theta$, so

$$\int \frac{\sec\theta\tan\theta}{\sqrt{\sec\theta+1}}\, d\theta = \int \frac{du}{\sqrt{u}} = 2u^{1/2} + C = 2\sqrt{\sec\theta+1} + C$$

Problem 9.

$$\int \ln x^3\, dx$$

Solution:

Method 1:

Use $f = \ln x^3, g' = 1$, so $f' = \frac{3x^2}{x^3} = \frac{3}{x}, g = x$. By IBP,

$$\int \ln x^3\, dx = x\ln x^3 - \int 3\, dx = x\ln x^3 - 3x + C$$

Method 2:

Simplify first, then

$$\int \ln x^3\, dx = \int 3\ln x\, dx = 3(x\ln x - x) + C$$

Problem 10.

$$\int \frac{1}{t^2 - 6t + 10}\, dt$$

Solution:

$$\int \frac{1}{t^2 - 6t + 10}\, dt = \int \frac{1}{(t-3)^2 + 1}\, dt = \arctan(t-3) + C$$

Problem 11.

$$\int_{-1}^{1} \frac{1}{x^3}\, dx$$

Solution: This is an improper integral of type II - you cannot use FTC II or that this is an odd function over a symmetric interval. Instead, the discontinuity is at 0, so

$$\int_{-1}^{1} \frac{1}{x^3}\, dx = \int_{-1}^{0} \frac{1}{x^3}\, dx + \int_{0}^{1} \frac{1}{x^3}\, dx$$

Considering the second integral,

$$\begin{aligned}\int_0^1 \frac{1}{x^3}\, dx &= \lim_{c\to 0^+} \int_c^1 \frac{1}{x^3}\, dx = \lim_{c\to 0^+} -\frac{1}{2x^2}\Big|_c^1 \\ &= \lim_{c\to 0^+} \left(\frac{1}{2c^2} - \frac{1}{2}\right) = \infty\end{aligned}$$

Therefore, the integral diverges.

Problem 12.

$$\int \frac{8x^2 - 2x + 3}{2x - 1}\, dx$$

Solution:

$$\begin{array}{r|l} & 4x + 1 \\ \hline 2x - 1 & 8x^2 - 2x + 3 \\ & \underline{-\,8x^2 + 4x} \\ & 2x + 3 \\ & \underline{-\,2x + 1} \\ & 4 \end{array}$$

$$\begin{aligned} \int \frac{8x^2 - 2x + 3}{2x - 1}\, dx &= \int \left(4x + 1 + \frac{4}{2x - 1}\right) dx \\ &= 2x^2 + x + 2\ln|2x - 1| + C \end{aligned}$$

Problem 13.

$$\int x^2 e^{2x}\, dx$$

Solution: Use $f = x^2, g' = e^{2x}$, so $f' = 2x, g = \frac{1}{2}e^{2x}$. By IBP,

$$\int x^2 e^{2x}\, dx = \frac{1}{2}x^2 e^{2x} - \int x e^{2x}\, dx$$

Use $f = x, g' = e^{2x}$, so $f' = 1, g = \frac{1}{2}e^{2x}$. By IBP again,

$$\begin{aligned} \int x^2 e^{2x}\, dx &= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{2}\int e^{2x}\, dx \\ &= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + C \end{aligned}$$

Problem 14.

$$\int_e^{e^2} \frac{1}{x \ln x}\, dx$$

Solution: Method 1: Using $u = \ln x$, $du = \frac{1}{x}\, dx$, $e \mapsto 1$, $e^2 \mapsto 2$, so

$$\int_e^{e^2} \frac{1}{x \ln x}\, dx = \int_1^2 \frac{1}{u}\, du = \ln|u|\Big|_1^2 = \ln 2$$

Method 2: Integrating directly,

$$\int_e^{e^2} \frac{1}{x \ln x}\, dx = \ln\ln x\Big|_e^{e^2} = \ln\ln e^2 - \ln\ln e = \ln 2$$

Problem 15.

$$\int_0^4 \sqrt{3x+4}\,dx$$

Solution:

Method 1: Using $u = 3x+4$, $du = 3\,dx$, $0 \mapsto 4$, $4 \mapsto 16$, so

$$\begin{aligned}\int_0^4 \sqrt{3x+4}\,dx &= \int_4^{16} \frac{1}{3}\sqrt{u}\,du = \frac{2}{9}u^{3/2}\Big|_4^{16} \\ &= \frac{2}{9}(16^{3/2} - 4^{3/2}) = \frac{2}{9}(4^3 - 2^3) = \frac{112}{9}\end{aligned}$$

Method 2: Integrating directly,

$$\begin{aligned}\int_0^4 \sqrt{3x+4}\,dx &= \frac{2}{9}\sqrt{3x+4}\Big|_0^4 = \frac{2}{9}(16^{3/2} - 4^{3/2}) \\ &= \frac{2}{9}(4^3 - 2^3) = \frac{112}{9}\end{aligned}$$

Problem 16.

$$\int \frac{x+1}{x(2x+1)}\,dx$$

Solution: If $\dfrac{x+1}{x(2x+1)} = \dfrac{A}{x} + \dfrac{B}{2x+1}$, the Heaviside cover-up method gives $A = \frac{0+1}{2(0)+1} = 1$ and $B = \frac{-\frac{1}{2}+1}{-\frac{1}{2}} = -1$, so

$$\begin{aligned}\int \frac{x+1}{x(2x+1)}\,dx &= \int \left(\frac{1}{x} + \frac{-1}{2x+1}\right)\,dx \\ &= \ln|x| - \frac{1}{2}\ln|2x+1| + C\end{aligned}$$

Problem 17.

$$\int_0^2 \frac{1}{\sqrt{2-x}}\,dx$$

Solution: The upper limit is a discontinuity, so

$$\begin{aligned}\int_0^2 \frac{1}{\sqrt{2-x}}\,dx &= \lim_{b\to 2^-}\int_0^b \frac{1}{\sqrt{2-x}}\,dx = \lim_{b\to 2^-}(-2\sqrt{2-x})\Big|_0^b \\ &= \lim_{b\to 2^-}(-2\sqrt{2-b} + 2\sqrt{2-0})) = 2\sqrt{2}\end{aligned}$$