Unit 1

A graphing calculator is required for these questions.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

1. The number of books sold in print of a new nonfiction book can be modeled by a function $N(t)$, given below, where t is the number of months since publication and h is a continuous function for $0 \leq t \leq 10$. Selected values of h are given in the table.

$$N(t) = \begin{cases} h(t), & 0 \leq t \leq 10 \\ \dfrac{8700t^2}{3t^2 + 100}, & t > 10 \end{cases}$$

t (months)	0	1	3	4	8	10
$h(t)$ (books)	0	455	780	1340	1915	2175

(a) Find the average rate of change of N between $t = 1$ and $t = 4$. Indicate units of measure.

(b) Must there be a time t, for $0 \leq t \leq 10$, when $N(t) = 1000$? Justify your answer.

(c) Find $\lim_{t \to \infty} N(t)$. Explain the meaning of $\lim_{t \to \infty} N(t)$ in the context of this problem.

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

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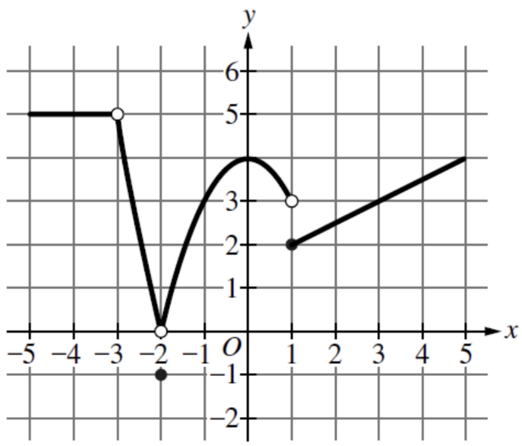
2. The graph of a function f is shown in the xy-plane for $-5 \le x \le 5$.



Let

$$g(x) = \begin{cases} 3\sin\left(\frac{\pi x}{4}\right) - 1, & x \le 2 \\ kx - 7, & x > 2 \end{cases}$$

for some constant k.

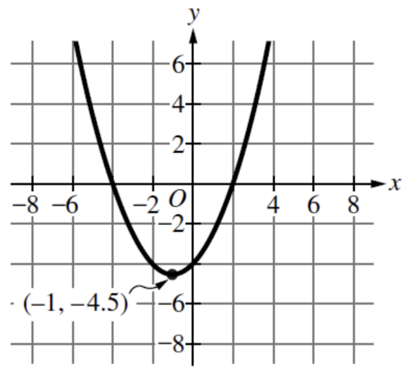
(a) For each of the values $a = -3$, $a = 0$, and $a = 1$, find $\lim\limits_{x \to a} f(x)$, or explain why it does not exist.

(b) Let $h(x) = 8f(x) - 5g(x)$. Find $\lim\limits_{x \to -2} h(x)$.

(c) Find the value of k so that g is continuous at $x = 2$. Justify your answer.

USE OF A CALCULATOR IS NOT ALLOWED FOR THIS QUESTION.

3. The graph of a quadratic function f is shown in the xy-coordinate plane.



(a) Which is greater: the average rate of change of f over the interval $[-3, 2]$ or the instantaneous rate of change of f at $x = 2$? Give a reason for your answer.

(b) Find $\lim\limits_{x \to 0^-} [-2f(x) + 9]$.

(c) Let $j(x) = \dfrac{x^2 - 7x + 10}{f(x)}$. Find all values of x where j has a discontinuity due to a vertical asymptote.

(d) Let $g(x) = \dfrac{f(x)}{x+4}$. Find $\lim\limits_{x \to -4} g(x)$.

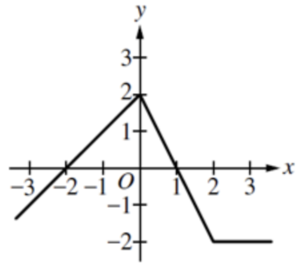
Unit 2

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

1.

x	$f(x)$	$f'(x)$
-2	2	-1
-1	0	0
0	1	1
1	2	$\frac{1}{2}$
2	4	2



Graph of $g(x)$

$f(x)$ and $g(x)$ are two continuous differentiable functions. Selected values of $f(x)$ and $f'(x)$ are given above. The graph of $g(x)$ is piecewise linear and made up of three line segments, as shown above.

(a) Find the average rate of change for $f(x)$ on the closed interval $[-2, 2]$.

(b) Estimate the instantaneous rate of change of $f(x)$ at $x = 1/2$.

(c) Find the average rate of change for $g(x)$ on the closed interval $[-2, 2]$.

(d) Find the instantaneous rate of change of $g(x)$ at $x = 1/2$.

(e) Find $\frac{d}{dx}[f(x) \cdot g(x)]_{x=1}$

(f) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

(g) Find $g'(-1)$ and $g'(5/2)$ or state if the values do not exist.

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

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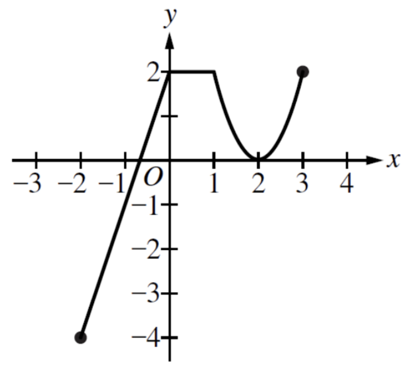
2.



Let $R(x) = x^2 + 3e^x$ and $S(x)$ be the continuous function shown, consisting of two line segments and the curve $y = 2(x-2)^2$ for $1 \le x \le 3$. (You may use a calculator, but for best practice, try to formulate your responses without one first—and then check yourself with your device!)

(a) Use the definition of the derivative to set up (but not evaluate) a limit that would find $R'(x)$ in terms of x.

(b) Find $\dfrac{d}{dx}[R(x) \cdot S(x)]_{x=2.5}$ to three decimal places.

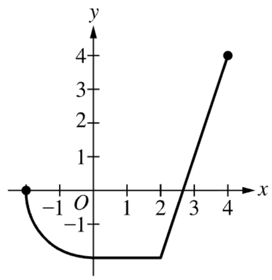
(c) Let $T(x) = \sin x \cdot R(x)$. Find $T'(5)$ to three decimal places.

(d) Let $U(x) = \dfrac{\tan x}{S(x)}$. Find $U'(x)$ and find $\left.\dfrac{dU}{dx}\right|_{x=\pi/4}$.

USE OF A CALCULATOR IS NOT ALLOWED FOR THIS QUESTION.

3.

x	$n(x)$	$n'(x)$
-2	-2	2
-1	1	1
$\frac{1}{2}$	2	1
1	$\frac{1}{2}$	-2
3	-1	2



Graph of p

$$r(x) = \begin{cases} 2x+4, & x < -1 \\ -x^3+1, & -1 \le x \le 1 \\ x^2 + \sin x, & x > 1 \end{cases}$$

Selected values of the twice differentiable function n are given in the table. The function p is the piecewise function made up of the quarter circle $y = \sqrt{4 - x^2}$ and two line segments, all as shown in the graph. A third function, r, is defined analytically as a piecewise function.

(a) Use continuity to show that $r(x)$ is not differentiable at $x = 1$.

(b) Use the definition of derivative with one-sided limits to show that $p'(2)$ does not exist.

(c) Estimate $n'(2)$.

(d) Find $\dfrac{d}{dx}[p(x) \cdot r(x)]$ for $0 < x < 1$ in terms of x.

(e) Let $Q(x) = \dfrac{p(x) \cdot r(x)}{n(x)}$. Find $Q'(\frac{1}{2})$.

Unit 3

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

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1.

x	$J(x)$	$J'(x)$
2	-5	8
3	-2	1
4	-1	3
5	4	7
6	8	2
7	9	6

(a) Let A be the function defined by $A(x) = J(3x+1)$. Find $A'(2)$.

(b) Let W be the inverse of J. Find $W'(4)$.

(c) Consider the curve given by the implicit relation $\sin y = \frac{J(x)}{10}$. It can be shown that the point $(2, \frac{7\pi}{6})$ lies on this curve. Find $\frac{dy}{dx}$ at the point $(2, \frac{7\pi}{6})$.

(d) The implicit relation in part (c) may be rewritten to give the explicit function $y = \arcsin\left(\frac{J(x)}{10}\right)$. If $u(x) = \frac{J(x)}{10}$, then $y = \arcsin u$. When $x = 2$ and $u(2) = -\frac{1}{2}$, find $\frac{dy}{du}$.

(e) Using the results from part (d), find $\frac{dy}{dx}$ when $x = 2$.

(f) Why are the answers to (c) and (e) different?

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

2. Consider the function $f(x) = 2(x^3 + 7)^{1/3} + x$.

   (a) Find the slope of the line tangent to the graph of f at the point where $x = 1$.

   (b) Write an expression for $f'(x)$. Use this expression to confirm your answer to part (a). Use a graphing calculator and your $f'(x)$ expression to evaluate the second derivative of f when $x = 1$.

   (c) Let $g(x)$ represent the inverse of $f(x)$. Write an equation for the line tangent to the graph of g at $x = 5$.

   (d) Consider the curve defined by the implicit relation $2y^2 - ye^x = 5x$. Find the slope of the line tangent to the curve at the point where $y = 4$.

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

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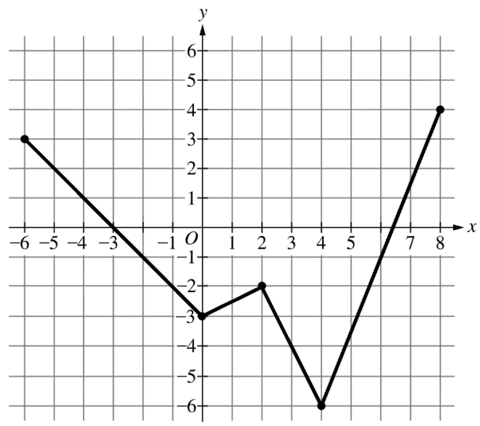
3.



Graph of h

Let g be the function defined by $g(x) = (x^2 + 2)^{\sin x}$. Let h be the function whose graph, consisting of four line segments, as shown in the figure.

(a) For some function $r(x)$, it is known that $r'(x) = \frac{h(x)}{2x+13}$. Find $r''(-5)$.

(b) Let q be the function defined by $q(x) = h(g(3x))$. Find $q'(0)$.

(c) Let k be the function defined by $k(x) = g(x) \cdot h(2x)$. Find $k'(3)$.

Unit 4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

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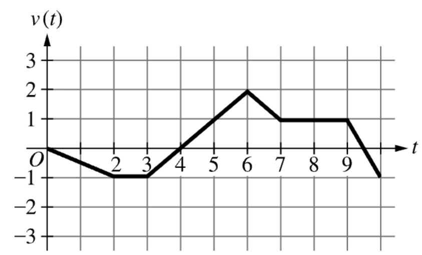
1.



A particle is moving along the x-axis with position $x(t)$ at time t, for $0 \leq t \leq 10$. At time $t = 6$, the particle is at position $x = -3$. The graph of the particle's velocity, v, as a function of time, t, is provided in the figure.

(a) Find the acceleration of the particle at time $t = 1$.

(b) Interpret the graph of $v(t)$ to identify all times on the interval $0 \leq t \leq 10$ when the particle changes direction.

(c) At time $t = 6$, is the particle moving towards the origin or away from the origin?

(d) Determine $\lim_{t\to 6}(t + 2 \cdot x(t))$ and $\lim_{t \to 6} \sin(\pi t)$. Evaluate $\lim_{t\to 6} \dfrac{t + 2 \cdot x(t)}{\sin(\pi t)}$.

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

2.

t (hours)	1	2	3	7	8
$M'(t)$ (shirts/hour)	70	82	86	115	121

Designs by AP makes and sells shirts. The number of shirts that have been produced at time t hours after the start of the work-day can be modeled by $M(t)$, where M is twice-differentiable and t is measured in hours for $0 \leq t \leq 8$. The table gives values of $M'(t)$, the rate of change in M, at selected times during the workday.

(a) Approximate $M''(t)$ using the average rate of change of $M'(t)$ over the interval. Using correct units, interpret the meaning of $M''(5)$ in the context of this problem.

(b) The total number of shirts that have been produced by time $t = 3$ is 225. Approximate $M(3.5)$ using the equation of the line tangent to the graph of M at $t = 3$. It is known that $M'(t)$ is increasing on $0 \leq t \leq 8$. Is the approximation an overestimate or underestimate of the total number of shirts produced at time $t = 3.5$?

(c) *Designs by AP* sells their shirts online. The number of shirts that have been sold online at time t hours after the start of the workday can be modeled by $S(t)$, where S is twice-differentiable and t is measured in hours for $0 \leq t \leq 8$. The rate at which shirts are sold, $S'(t)$, is given by the equation $S'(t) = 20\sqrt{t} - t^2 + 55 \cdot \arctan(t^3)$. Determine all time intervals, on $0 \leq t \leq 8$, for which the rate of sales is at least 100 shirts per hour.

(d) Calculate $M'(3) - S'(3)$. Using correct units, interpret the meaning of $M'(3) - S'(3)$ in the context of this problem.

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

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3. Ayan is driving due east and passes through an intersection at time $t = 0$ seconds. The velocity of Ayan's car can be modeled by the equation $v(t) = -\frac{1}{10}t^3 + \frac{3}{2}t^2 - 4t + 10$, for $0 \leq t \leq 11$, where $v(t)$ is measured in meters per second and t is measured in seconds. Ayan's car is 900 meters east of the intersection after 10 seconds.

   (a) Find the acceleration of Ayan's car at time $t = 10$. Indicate units of measure.

   (b) Is Ayan's car speeding up or slowing down at time $t = 10$?

   (c) Keng is standing 1200 meters due north of the intersection. Find the rate of change of the distance between Keng and Ayan, in meters per second, at time $t = 10$.

   (d) Ben is in a hot air balloon directly over the intersection. Let θ be the angle of elevation from Ayan to Ben, where θ is measured in radians. Let A be the distance between Ayan's car and the intersection and B be the distance between Ben and the intersection at time t, both measured in meters. These quantities satisfy the relationship $\tan\theta = \frac{B}{A}$. Write an equation that describes the relationship between the rates of change of θ, A, and B.

   (e) Using θ as defined in Part D, at time $t = 10$, $\frac{d\theta}{dt} = -0.0073$. Using correct units, interpret the meaning of $\frac{d\theta}{dt}$ in the context of this problem.