## Arithmetic Sequences Part II (Grade 6)

Sometimes, terms of an arithmetic sequence are not explicitly given.

**Example 1.** At an arcade, a birthday party costs \$120 to book a slot and an additional \$10 per person in the party.

(a) How much money does it cost to book a party for 6 people?

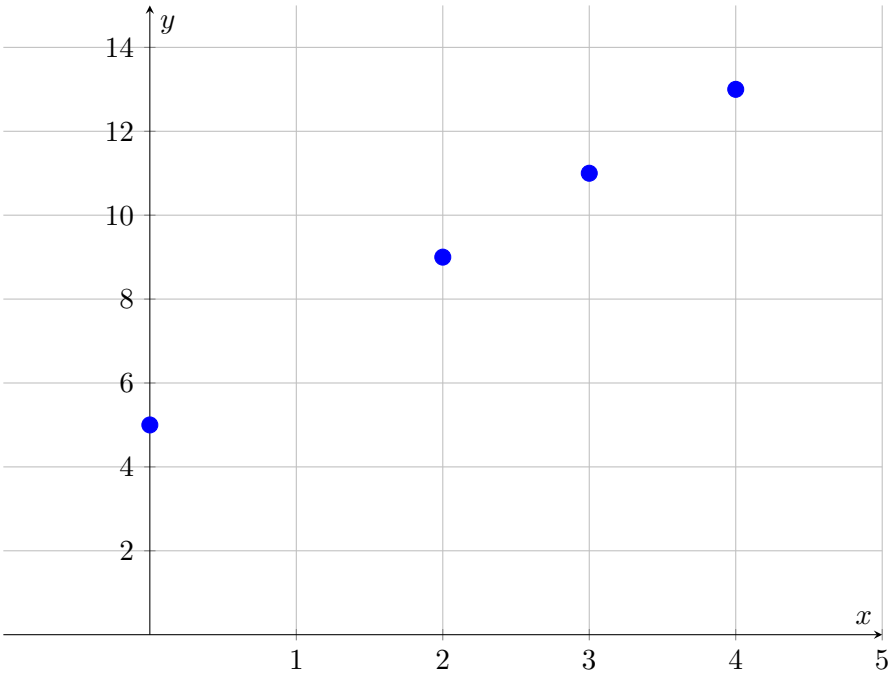
(b) If $P$ is the price of a reservation in dollars and $n$ is the number of people for the party, find a formula for $P$ in terms of $n$.

**Solution 1.** (a) \$180

(b) Since each person costs \$10 and there are $n$ people, the cost for these people is $10n$. However, there is also the fixed cost of \$120 for the reservation, so we need to add that to the overall price. We get

$$P = 10n + 120$$

**Example 2.** Consider the following data.




(a) What $y$-value is most likely corresponding to the $x$-value of 1?

(b) What is the most likely general formula for the following data?

**Solution 2.** (a) Missing $(1, 7)$.

(b) The $y$-values are increasing by 2 when $x$ increases by 1, so the formula is of the form $y = 2x+b$ for some $b$. Notice that at $x = 0$, $y = 5$, so $b = 5$. We get

$$y = 2x + 5$$

(Check that this works for the pattern!)

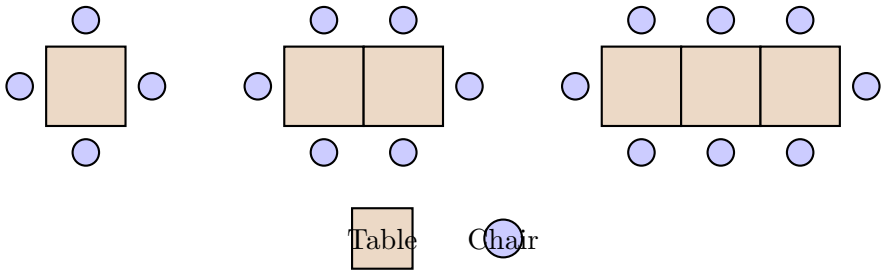
Note: this is also known as a *linear equation*, and the number 2 represents the *slope* while the number 5 represents the *y-intercept*. Do you see why?

**Example 3.** In a restaurant, square tables are used. Each edge of the table can fit 1 chair. When more than 4 chairs are needed, they put together multiple square tables in a row and place chairs on the open sides.

(a) How many chairs are used for an arrangement of 8 square tables?

(b) How many square tables are needed for a party of 16 people?

(c) If $C$ is the number of chairs and $T$ is the number of square tables, what is a formula for $C$ in terms of $T$?

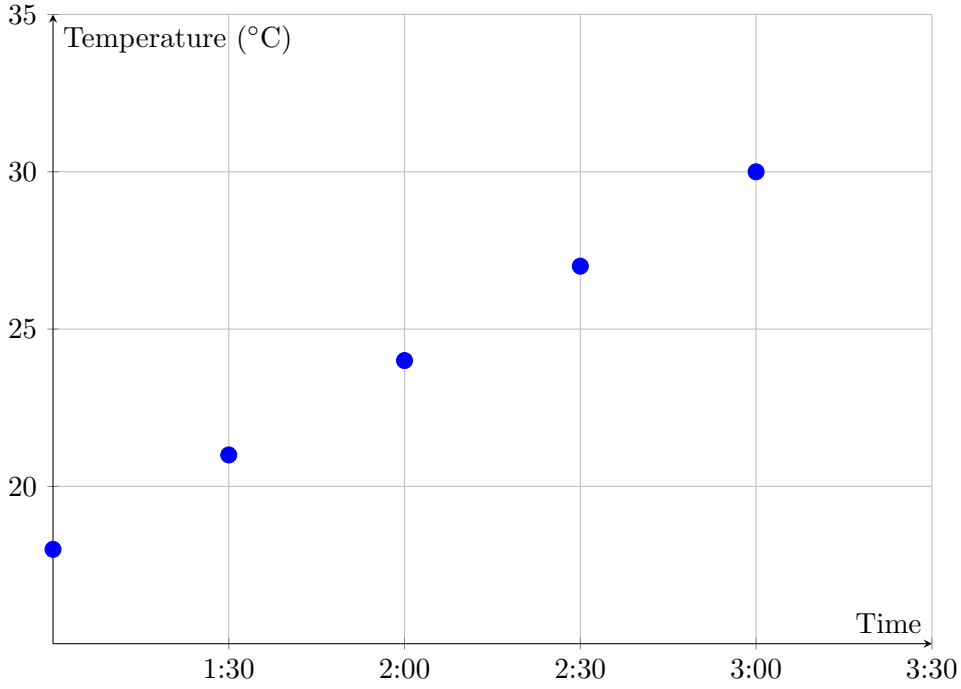



**Solution 3.** (a) 18 chairs (either use the general formula or work out the terms until this point)

(b) 7 square tables (either use the general formula or work out the terms until this point)

(c) The number of chairs increases by 2 every time we add a table, so the formula must be of the form $C = 2T + b$ for some $b$. When $T = 1$, we have $C = 4$, so $4 = 2(1) + b$ gives $b = 2$. Therefore, $C = 2T + 2$.

1. The graph below shows the temperature recorded every 30 minutes starting at 1:00 PM.




   At approximately what time does the temperature reach 30°C?

2. A carpenter charges a $50 call-out fee plus $30 per hour of work.
   (a) How much does it cost for 2 hours of work? For 5 hours?
   (b) Find a formula for the cost $C$ in dollars in terms of the number of hours $h$.
   (c) If a customer was charged $230, how many hours did the carpenter work?

3. Matchsticks are used to make a row of equilateral triangles.

1 triangle 2 triangles 3 triangles

(a) How many matchsticks are needed for 1 triangle? For 2 triangles? For 3 triangles?

(b) How many matchsticks are needed for a row of 10 triangles?

(c) Find a formula for the number of matchsticks $M$ in terms of the number of triangles $t$.

4. Water is being pumped into a tank. The table below shows the water level at different times.

| Time (minutes) | Water level (cm) |
|---|---|
| 2 | 14 |
| 4 | 23 |
| 6 | 32 |
| 8 | 41 |

(a) What is the common difference? What does it represent in this context?

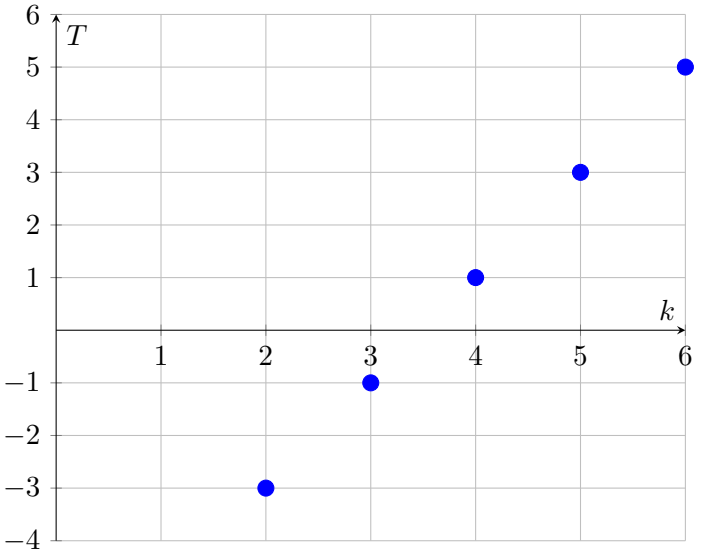
(b) What was the water level at time 0 minutes (before pumping started)?

(c) Find a formula for the water level $L$ in terms of time $t$.

5. A phone plan costs \$20 per month plus \$0.10 per text message.
   (a) How much does it cost if you send 50 texts in a month? 100 texts?
   (b) Find a formula for the monthly cost $C$ in dollars in terms of the number of texts $n$.
   (c) If your bill was \$35, how many texts did you send?

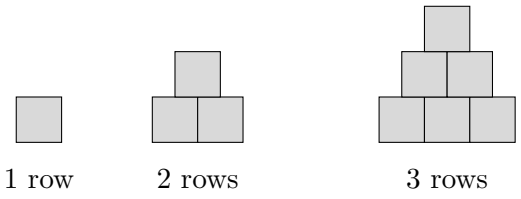
6. A taxi company charges a flat fee of \$2 plus \$3 per kilometre driven.
   (a) How much would a 5 kilometre trip cost?
   (b) If $d$ is the distance driven in kilometres and $f$ is the fee charged in dollars, write a formula for $f$ in terms of $d$.
   (c) A customer paid \$32. How many kilometres did they travel?

7. Consider the graph below showing an arithmetic sequence:




What is a general formula for $T$ in terms of $k$?

8. Cans are stacked in a triangular pyramid. The bottom row has 1 can, the next row up has 2 cans, the third row has 3 cans, and so on. (This is a triangular number pattern.)




(a) How many cans are in a pyramid with 1 row? With 2 rows? With 3 rows?

(b) (*) How many cans are in a pyramid with 20 rows?