Algebra (solving equations)

To solve an equation for an unknown variable, we isolate the desired variable step-by-step. One of the easiest ways to do this is to undo the order of operations: for instance, addition and subtraction are the last to perform, so we undo addition with subtraction first.

Example 1.

$$\begin{aligned} 4x + 1 &= 21 \\ 4x + 1 - 1 &= 21 - 1 \\ 4x &= 20 \\ \frac{4x}{4} &= \frac{20}{4} \\ x &= 5 \end{aligned}$$

In practice, we typically skip the 2nd and 4th step, as it is clear what we did. The key idea when it comes to solving equations is that you **must perform the same operation to both sides of the equation**.

The distributive property says that

$$a(b + c) = ab + ac$$

for any real numbers a, b, c. This allows us to simplify certain equations in order to solve them. Note that unless otherwise specified, the preferred form of an answer is as a simplified, improper fraction.

Example 2.

$$\begin{aligned} 2(x + 1) + 3(2x - 1) &= 9 \\ 2x + 2 + 6x - 3 &= 9 \\ 8x - 1 &= 9 \\ 8x &= 12 \\ x &= \frac{12}{8} = \frac{3}{2} \end{aligned}$$

Be careful about negatives! I recommend grouping the sign with the coefficient so that you don't make a silly mistake. Remember that the product of two negatives is positive.

Example 3.

$$3x - 2(x - 1) = 3x + (-2)(x - 1) = 3x + (-2)(x) + (-2)(-1) = 3x - 2x + 2$$

NOT

$$3x - 2(x - 1) = 3x - 2(x) + 2(-1) = 3x - 2 - 1$$

When dealing with fractions, you have two main options: distribute as usual and solve with possibly fractional coefficients, or (my personal choice) clear the denominator by multiplying by the least common multiple of the denominators.

Example 4.

$$\frac{3x-1}{4} - \frac{6-x}{6} = \frac{2x-1}{3}$$

Method 1:

$$\begin{aligned}
\frac{3x-1}{4} - \frac{6-x}{6} &= \frac{2x-1}{3} \\
\frac{3}{4}x - \frac{1}{4} - 1 + \frac{x}{6} &= \frac{2}{3}x - \frac{1}{3} \\
\left(\frac{3}{4} + \frac{1}{6} - \frac{2}{3}\right)x &= \frac{1}{4} + 1 - \frac{1}{3} \\
\frac{1}{4}x &= \frac{11}{12} \\
x &= \frac{11}{3}
\end{aligned}$$

Method 2:

$$\begin{aligned}
\frac{3x-1}{4} - \frac{6-x}{6} &= \frac{2x-1}{3} \\
3(3x-1) - 2(6-x) &= 4(2x-1) \\
9x - 3 - 12 + 2x &= 8x - 4 \\
3x &= 11 \\
x &= \frac{11}{3}
\end{aligned}$$

Notice it's easiest to simplify the fraction before moving on, so that

$$12 \cdot \frac{3x-1}{4} = 3(3x-1) \quad \text{NOT} \quad 12 \cdot \frac{3x-1}{4} = \frac{36x-12}{4}$$

Be careful not to multiply a single term more times than you are supposed to! A single term is composed of factors that are multiplied, so multiplying a term by a number means only one of the factors is multiplied (unless you break down the number into several smaller divisors).

$$2 \cdot \frac{3}{2}\left(\frac{1}{2}x - 1\right) = 3\left(\frac{1}{2}x - 1\right) \quad \text{NOT} \quad 2 \cdot \frac{3}{2}\left(\frac{1}{2}x - 1\right) = 3(x-2)$$

1. Solve the following equations for x:

   (a) (1 point)

   $$4x + 7 = 23$$

   (b) (1 point)

   $$7x - 9 = 12$$

   (c) (1 point)

   $$5x + 8 = 2x + 20$$

2. Solve the following equations for x:

   (a) (1 point)

   $$3x - 5 = 16$$

   (b) (1 point)

   $$2x + 11 = 5x - 4$$

   (c) (1 point)

   $$6x + 9 = 3x + 18$$

3. Solve the following equations for x:

(a) (1 point)

$$8x - 7 = 3x + 18$$

(b) (1 point)

$$10 - 2x = 4x + 14$$

(c) (1 point)

$$7x + 5 = 2x + 20$$

4. Solve the following equations for x:

   (a) (1 point)

$$3(2x + 4) - 5 = 19$$

   (b) (1 point)

$$2(3x - 5) + 4(x + 1) = 18$$

   (c) (1 point)

$$5(x + 2) - 3(2x - 1) = 7$$

5. Solve the following equations for x:

(a) (1 point)

$$4(2x - 3) + 3(1 - x) = 10$$

(b) (1 point)

$$2(5x + 1) - 4(x - 2) = 6$$

(c) (1 point)

$$3(x - 2) + 2(2x + 1) = 11$$

6. Solve the following equations for x:

(a) (1 point)

$$2(3x+1)-5(x-2)=4$$

(b) (1 point)

$$4(x-3)-2(2x+1)=6$$

(c) (1 point)

$$2(x+3)-(x-5)=x+11$$

7. Solve the following equations for x:

   (a) (1 point)

$$\frac{3x+2}{4} - \frac{x-1}{2} = 3$$

   (b) (1 point)

$$\frac{2x-3}{5} + \frac{x+4}{3} = 7$$

8. Solve the following equations for x:

(a) (1 point)

$$\frac{3x-1}{4} + \frac{2x+3}{5} = \frac{7x+2}{6}$$

(b) (1 point)

$$\frac{5x+2}{3} - \frac{x-1}{2} = \frac{7x+5}{6}$$

9. Solve the following equations for x:

(a) (1 point)

$$\frac{2x+3}{5} + \frac{3x-4}{6} = \frac{5x+1}{4}$$

(b) (1 point)

$$\frac{4x-5}{7} - \frac{3x+2}{5} = \frac{2x-3}{3}$$

10. Solve the following equations for x:

(a) (1 point)

$$7(2x-3)-4(x+5)=3(2x+1)+8$$

(b) (1 point)

$$5(x-2)-3(2x+1)+4=2(3x-4)-1$$

11. Solve the following equations for x:

(a) (1 point)

$$\frac{3x+5}{4} + \frac{2x-3}{6} - \frac{x+1}{3} = \frac{5x-2}{12}$$

(b) (1 point)

$$\frac{5x-2}{3} - \frac{2x+1}{4} = \frac{3x-7}{6} + \frac{1}{2}$$

12. Solve the following equations for x:

(a) (1 point)

$$2(3x-1) - \frac{4x+5}{3} = 5x - 7 + \frac{x-2}{2}$$

(b) (1 point)

$$\frac{3x-2}{5} + 2(2x+1) = \frac{7x+4}{2} - \frac{x-3}{10}$$

13. Solve the following equation for x:

$$\frac{2x+3}{4} - 3\left(\frac{x-1}{2} - 2\right) = \frac{5x-7}{6} + 2\left(\frac{x+2}{3} - 1\right)$$

14.

$$\frac{\frac{3x}{5}+2}{\frac{1}{2}}-\frac{4-\frac{x}{3}}{\frac{2}{3}}=\frac{\frac{5x}{6}-1}{2}+\frac{3}{4}$$