Arithmetic Sequences Part I (Grade 6)

In Grade 6, a common pattern that is encountered is one in which the terms increase or decrease by a fixed amount. This is known as an *arithmetic sequence*, and the fixed change is called the *common difference.*

Example 1.

$$2, 5, 8, 11, \ldots$$

In this pattern, terms increase by 3.

Example 2. What is the 6th term of this pattern?

$$5, 3, 1, -1, \ldots$$

In this pattern, terms decrease by 2. The next few terms are: $-3, -5, -7, \ldots$ The 6th term is -5.

We can use a variable to denote the term number, such as n or x(sometimes called the *index*). We may use another variable indexed by n to denote the nth term, such as a_n or t_n. Sometimes, we may simply use x as the term number and y as the result.

Example 3.

$$2, 5, 8, 11, \ldots$$

In this pattern, $t_1 = 2, t_2 = 5, t_3 = 8, t_4 = 11, \ldots$

As a table:

n	t_n
1	2
2	5
3	8
4	11

Example 4.

$$5, 3, 1, -1, \ldots$$

As a table:

x	y
1	5
2	3
3	1
4	-1

The general formula for any arithmetic sequence looks something like $t_n = mn+b$. To find it, notice that as the term number increases by 1, the value of the term changes by the common difference. Then the coefficient of the variable for term number must be the common difference. From here, we can "shift" until our formula agrees with the sequence, or simply substitute known values and use algebra to solve for an unknown value.

Example 5. What is the formula for the general term?

n	t_n
1	2
2	5
3	8
4	11

The common difference is 3, so our formula should be of the form $t_n = 3n + b$. Look what happens if we try simple $t_n = 3n$:

n	t_n	$3n$
1	2	3
2	5	6
3	8	9
4	11	12

All of our answers are too big by 1. The correct formula is $t_n = 3n - 1$.

We can use the general formula to help in finding specific values without having to make lots of computations.

Example 6. What is the 1000th term in this pattern?

$$5, 3, 1, -1, \ldots$$

The common difference is -2, so our formula should be of the form $y = -2x + b$. When $x = 1$, we get $y = 5$, so $5 = -2(1) + b$ gives $b = 7$. The correct formula is $y = -2x + 7$.

When $x = 1000$, we get $y = -2(1000) + 7 = -1993$.

1. Find the common difference for these arithmetic sequences:

   (a)

$$8, 3, -2, \ldots$$

   (b)

$$-3, 3, 9, \ldots$$

   (c)

$$\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{3}{2}, \ldots$$

2. Find the missing term in these arithmetic sequences:

   (a)

$$6, 10, \_, 18, 22, \ldots$$

   (b)

$$27, \_, 21, 18, \ldots$$

   (c)

$$\frac{5}{2}, \frac{11}{4}, \_, \frac{13}{4}$$

3. Find a formula for the nth term for each arithmetic sequence:

(a)

$$7, 11, 15, 19, \ldots$$

(b)

$$24, 20, 16, 12, \ldots$$

(c)

m	b_m
1	8
2	5
3	2
4	-1

(d)

x	y
0	2
1	7
2	12
3	17

4. Find the indicated term for each arithmetic sequence:

(a) Find the 21st term: $7, 11, 15, 19, \ldots$

(b) Find the 18th term: $24, 20, 16, 12, \ldots$

(c) Find the 30th term using the table:

k	t_k
1	8
2	5
3	2
4	-1

(d) Find the value of C when $n = 19$:

n	C
0	2
1	7
2	12
3	17

5. Challenge problems:

   (a) The 5th term of an arithmetic sequence is 23, and the 9th term is 39. Find the 20th term.

   (b) The 3rd term of an arithmetic sequence is 20, and the 7th term is 8. Find the 15th term.

   (c) In an arithmetic sequence, $t_3 = 7$ and $t_7 = 10$. Find t_{20}.