Mini-math Div 3/4: Wednesday, December 3, 2025 (8.7-8.13) - 18 minutes

SOLUTIONS

1. (2 points) The base of a solid is the region bounded by $y = x^{1/3}$, $y = 3$, and $x = 1$. Cross-sections perpendicular to the x-axis are rectangles whose heights are twice their base. Set up an integral (or integrals) that represents the volume of the solid.

Solution:

$$\int_1^{27} 2(3 - x^{1/3})^2 \, dx$$

2. (2 points) Consider the region R bounded by $y = \arctan x$, $x = -1$, and $y = 1$. Find an integral (or integrals) that represents the volume of the solid of revolution if we revolve the region R about the line $x = -1$.

Solution:

$$\pi \int_{-\pi/4}^{1} (\arctan y + 1)^2 \, dy$$

3. (2 points) Set up an integral (or integrals) that represents the volume of the solid generated by revolving the region above $y = x^3$, below the line $y = 8$, and between $x = 0$ and $x = 2$ around the x-axis.

Solution:

$$\pi \int_0^2 \left[8^2 - (x^3)^2\right] \, dx$$

4. (2 points) Consider the region R that is bounded by $y^2 = 7x + 8$ and $y = x + 2$. If R is the base of a solid and cross-sections perpendicular to the y-axis are semi-circles, set up an integral (or integrals) that represents the volume of the solid.

Solution:

$$\int_1^6 \frac{\pi}{2}\left(\frac{(y-2) - \frac{1}{7}(y^2-8)/2}{2}\right)^2 dy$$

5. (2 points) Let R be the region enclosed by $y = x^2 - 4$ and $y = 2x + 4$. Find an integral (or integrals) that represents the perimeter of the region R.

Solution:

$$\int_{-2}^4 \sqrt{1+(2)^2}\,dx + \int_{-2}^4 \sqrt{1+(2x)^2}\,dx \quad \text{or} \quad 6\sqrt{5} + \int_{-2}^4 \sqrt{1+4x^2}\,dx$$

6. (2 points) Let R be the region in the first quadrant below $y = 4 - x^2$. Find an integral (or integrals) that represents the volume of the solid of revolution if we revolve the region R about the line $y = 4$.

Solution:

$$\pi \int_0^2 \left[4^2 - (4-(4-x^2))^2\right] dx = \pi \int_0^2 \left[4^2 - x^4\right] dx$$