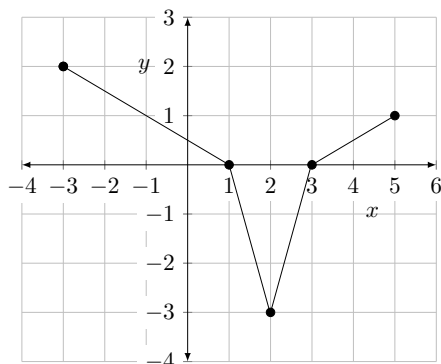


SOLUTIONS

1. (2 points) The graph of the piecewise linear function f is shown in the figure to the right. What is the average value of f over $[-3, 5]$?

- A. -1
- B. $-1/8$
- C. 0
- D. $1/4$**
- E. 2



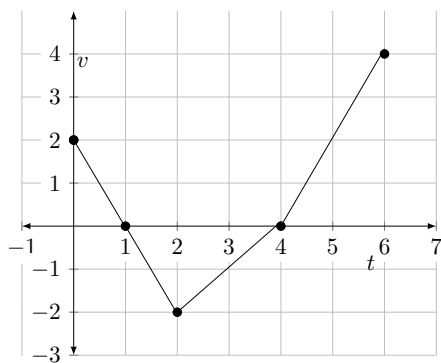
Solution:

$$f_{avg} = \frac{\int_{-3}^5 f \, dx}{5 - (-3)} = \frac{4 - 3 + 1}{8} = \frac{1}{4}$$

D is correct.

2. (2 points) The graph of the velocity of a function is the piecewise linear function shown in the figure to the right. The initial position of the particle at time $t = 0$ is $x = 1$. What is the total distance the particle travels from $t = 0$ to $t = 6$?

- A. 2
- B. 3
- C. 4
- D. 8**
- E. 9



Solution:

$$\int_0^6 |v(t)| \, dt = 1 + 3 + 4 = 8$$

D is correct.

3. (2 points) The acceleration of a particle is modelled by $a(t) = 2t + 3$ for $t \geq 0$. At $t = 0$, the velocity of the particle is -2 and its position is 2.5 . What is the change in displacement of the particle from $t = 0$ to $t = 3$?

A. 9 B. 16 **C. 16.5** D. 19 E. 22.5

Solution:

$$v(t) = \int a(t) dt = \int (2t + 3) dt = t^2 + 3t + C$$

Since $v(0) = -2$, we know $C = -2$. Then the change in displacement is

$$\Delta x = \int_0^3 v(t) dt = \int_0^3 (t^2 + 3t - 2) dt = \left(\frac{1}{3}t^3 + \frac{3}{2}t^2 - 2t \right) \Big|_0^3 = 9 + \frac{27}{2} - 6 = 16.5$$

C is correct.

4. (2 points) Suppose f is a differentiable function. Which of the following statements are true:

- (I) The average value of the derivative of f over $[a, b]$ is the same as the average rate of change of f over $[a, b]$.
 (II) There exists a $c \in [a, b]$ for which $f(c)$ equals the average value of f over $[a, b]$.

A. (I) only B. (II) only **C. Both (I) and (II)** D. Neither (I) nor (II)
 E. The truth of both statements depend on the specific choice of f

Solution: By FTC II,

$$\frac{\int_a^b f'(x) dx}{b - a} = \frac{f(b) - f(a)}{b - a}$$

so (I) is true.

Since f is continuous and $[a, b]$ is a closed and bounded interval, the Extreme Value Theorem tells us that there are values x_{\min} and x_{\max} for which $f(x_{\min}) \leq f(x) \leq f(x_{\max})$ for all $x \in [a, b]$. Then

$$\frac{\int_a^b f(x_{\min}) dx}{b - a} \leq \frac{\int_a^b f(x) dx}{b - a} \leq \frac{\int_a^b f(x_{\max}) dx}{b - a}$$

$$f(x_{\min}) \leq f_{\text{avg}} \leq f(x_{\max})$$

By the Intermediate Value Theorem, there exists $c \in [a, b]$ (between x_{\min} and x_{\max}) such that $f(c) = f_{\text{avg}}$, so (II) is true.

C is correct.

5. (2 points) Water is leaking out of a tub at a rate modelled by $r(t) = \frac{1}{t^2 + 1} \text{cm}^3/\text{min}$, where t is in minutes. If the initial volume of the tub is $160\,000 \text{ cm}^3$, which of the following represents the volume of the tub at time t ?

A. $160000 + \int_0^t r(x) dx$

B. $160000 - \int_0^t r(x) dx$

C. $160000 - \frac{1}{t^2 + 1}$

D. $160000 + \frac{r(t)}{t^2 + 1}$

E. $\frac{1}{t^2 + 1}$

Solution: By the Net Change Theorem,

$$V(t) = 160000 - \int_0^t r(x) dx$$

B is correct.

6. (2 points) Find the area of the bounded region below both $y = x^2$ and $y = 2 - x$ and above the x -axis.

A. $2/3$

B. **$5/6$**

C. 1

D. $7/6$

E. 3

Solution: Integrating with respect to y ,

$$A = \int_0^1 [(2 - y) - \sqrt{y}] dy = \left(2y - \frac{y^2}{2} - \frac{2}{3}y^{3/2} \right) \Big|_0^1 = 2 - \frac{1}{2} - \frac{2}{3} = \frac{5}{6}$$

OR Integrating with respect to x (with 2 regions),

$$A = \int_0^1 x^2 dx + \int_1^2 (2 - x) dx = \frac{x^3}{3} \Big|_0^1 + \left(2x - \frac{x^2}{2} \right) \Big|_1^2 = \frac{1}{3} + 4 - 2 - 2 + \frac{1}{2} = \frac{5}{6}$$

B is correct.

7. (2 points) Set up integral(s) with respect to y that represents the area bounded by $y = 2x^{1/3}$, $y = 4$, and $x = 1$.

Solution: The right function is $x = (y/2)^3$ and the left function is $x = 1$. We integrate from $y = 2$ to 4, since $y = 2x^{1/3}$ intersects $x = 1$ at $y = 2$:

$$A = \int_2^4 \left[\left(\frac{y}{2} \right)^3 - 1 \right] dy$$