**Mini-math Div 3/4: Friday, September 12, 2025 (6.1-6.3) - 6 minutes**

**SOLUTIONS**

1. (1 point) Choose the limit of the Riemann Sum that is the integral: $\displaystyle\int_2^4 \frac{1}{x+2}\,dx$

A. $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{\frac{k}{n}+2}\cdot\left(\frac{2}{n}\right)$

B. $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{\frac{2k}{n}+2}\cdot\left(\frac{2}{n}\right)$

C. $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{\frac{k}{n}+4}\cdot\left(\frac{2}{n}\right)$

D. $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{\frac{2k}{n}+4}\cdot\left(\frac{2}{n}\right)$

**Solution:** $\Delta x = \frac{4-2}{n} = \frac{2}{n}$, and $x_k = a + \Delta x = 2 + \frac{2k}{n}$, so if $f(x) = \frac{1}{x+2}$, we have $f(x_k) = \frac{1}{2+\frac{2k}{n}+2} = \frac{1}{\frac{2k}{n}+4}$.

(d) is correct.

2. (1 point) Choose the integral that is the limit of the Riemann Sum: $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \sin\left(1+\frac{8k}{n}\right)\cdot\frac{4}{n}$

A. $\displaystyle\int_0^4 \sin(1+2x)\,dx$ B. $\displaystyle\int_1^5 \sin(1+x)\,dx$ C. $\displaystyle\int_1^5 \sin(1+2x)\,dx$ D. $\displaystyle\int_1^5 \sin x\,dx$

**Solution:**

$$\lim_{n\to\infty}\sum_{k=1}^{n} \sin\left(1+\frac{8k}{n}\right)\cdot\frac{4}{n} = \lim_{n\to\infty}\sum_{k=1}^{n} \sin\left(1+2\cdot\frac{4k}{n}\right)\cdot\frac{4}{n}$$

(a) is correct.

3. (1 point) Suppose $f$ is a concave up function and the following are selected values of $f$:

| $x$ | 0 | 1 | 3 | 4 | 6 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 2 | 4 | 6 | 12 |

If we use the trapezoidal rule with 4 unequal subintervals to approximate $\int_0^6 f(x)\,dx$, then:

A. $\int_0^6 f(x)\,dx \approx 31.5$ and this is an underestimate

B. $\int_0^6 f(x)\,dx \approx 31.5$ and this is an overestimate

C. $\int_0^6 f(x)\,dx \approx 63$ and this is an underestimate

D. $\int_0^6 f(x)\,dx \approx 63$ and this is an overestimate

**Solution:**

$$(3+2)/2 \cdot 1 + (2+4)/2 \cdot 2 + (4+6)/2 \cdot 1 + (6+12)/2 \cdot 2 = \frac{63}{2} = 31.5$$

Since $f$ is concave up, the trapezoidal rule is an overestimate.

(b) is correct.