Renert School: Series Bee 2025–2026

Problem 1. If the nth partial sum of a series $\sum_{k=1}^{\infty} a_k$ is

$$S_n = \frac{3n+2}{2n-3}$$

find a_4.

Problem 2. If the Nth partial sum of a series $\sum_{n=2}^{\infty} a_n$ is

$$S_N = \frac{(\ln N)^2}{N}$$

find $\displaystyle\sum_{n=2}^{\infty} a_n$.

Problem 3. Find the sum of the infinite series

$$5 - \frac{5}{3} + \frac{5}{9} - \frac{5}{27} + \cdots$$

Problem 4. Express $2.0\overline{63}$ as a ratio of two integers in lowest terms.

Problem 5. Is the series $S = \displaystyle\sum_{n=1}^{\infty} 3^{3n} 5^{2-2n}$ convergent or divergent? If convergent, find its sum.

Problem 6. Find the sum of the series

$$\sum_{n=2}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right)$$

Problem 7. If $\displaystyle\sum_{n=1}^{\infty} a_n = 2$ and $\displaystyle\sum_{n=1}^{\infty} b_n = -3$, find the sum of the infinite series $\displaystyle\sum_{n=1}^{\infty} (5a_n - 2b_n)$

Problem 8. Find the value of p so that the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ is convergent.

Problem 9. Does the following series converge or diverge, and why?

$$\sum_{n=2}^{\infty} \frac{3}{n^2 - 6n}$$

Problem 10. Does the following series converge or diverge, and why?

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

Problem 11. Does the following series converge or diverge, and why?

$$\sum \frac{2^{n+1}-1}{2^{n+3}}$$

Problem 12. Does the following series converge or diverge, and why?

$$\sum_{n=1}^{\infty} n^2 e^{-2n}$$

Problem 13. Does the following series converge or diverge, and why?

$$\sum_{n=1}^{\infty} \frac{3+\sin n}{n^2}$$

Problem 14. Does the following series converge or diverge, and why?

$$\sum_{n=1}^{\infty} \frac{3+\sin n}{n}$$

Problem 15. Does the following series converge or diverge, and why?

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$$

Problem 16. Does the following series converge or diverge, and why?

$$\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$$

Problem 17. Does the following series converge or diverge, and why?

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

Problem 18. Does the following series converge or diverge, and why?

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{e^{1/n}}{\sqrt{n}}$$

Problem 19. Does the following series converge absolutely, converge conditionally, or diverge, and why?

$$\sum_{n=1}^{\infty} (-1)^n \frac{\cos(\pi n)}{n}$$

Problem 20. Does the following series converge absolutely, converge conditionally, or diverge, and why?

$$\frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \frac{4}{17} + \cdots$$

where in the alternating sum, the numerators are increasing by 1 and the denominators are increasing by the next odd number.

Problem 21. For what value(s) of x, if any, will the following series conditionally converge?

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n3^n}$$