Name: ────────────────────   Mark: ────────────────────

Algebra (solving equations)

To solve an equation for an unknown variable, we isolate the desired variable step-by-step. One of the easiest ways to do this is to undo the order of operations: for instance, addition and subtraction are the last to perform, so we undo addition with subtraction first.

Example 1.

$$4x + 1 = 21$$
$$4x + 1 - 1 = 21 - 1$$
$$4x = 20$$
$$\frac{4x}{4} = \frac{20}{4}$$
$$x = 5$$

In practice, we typically skip the 2nd and 4th step, as it is clear what we did. The key idea when it comes to solving equations is that you **must perform the same operation to both sides of the equation**.

The distributive property says that
$$a(b + c) = ab + ac$$
for any real numbers a, b, c. This allows us to simplify certain equations in order to solve them.

Example 2.

$$2(x + 1) + 3(2x - 1) = 3$$
$$2x + 2 + 6x - 3 = 3$$
$$8x - 1 = 3$$
$$8x = 4$$
$$x = \frac{4}{8} = \frac{1}{2}$$

Be careful about negatives! I recommend grouping the sign with the coefficient so that you don't make a silly mistake. Remember that the product of two negatives is positive.

Example 3.

$$3x - 2(x - 1) = 3x + (-2)(x - 1) = 3x + (-2)(x) + (-2)(-1) = 3x - 2x + 2$$

NOT

$$3x - 2(x - 1) = 3x - 2(x) + 2(-1) = 3x - 2 - 1$$

Final note: Unless otherwise specified, the preferred form of an answer is as a simplified, improper fraction.

1. Solve the following equations for x:

   (a) (1 point)
   $$6x - 7 = 43$$

   (b) (1 point)
   $$2x + 31 = 3$$

   (c) (1 point)
   $$9x + 28 = 3x - 5$$

2. Solve the following equations for x:

   (a) (1 point)
   $$3(2x + 3) + 2(x - 5) = -19$$

   (b) (1 point)
   $$5(2x - 3) - 6(x + 1) = 39$$

   (c) (1 point)
   $$2(3x - 7) - 6(3 - 4x) = -17$$

Note: For fractions, it's easiest to clear the denominators by multiplying by the least common denominator first.

3. Solve the following equations for x:

   (a) (1 point)
   $$\frac{2x + 7}{4} - \frac{x - 4}{3} = x + 2$$

   (b) (1 point)
   $$\frac{x - 2}{4} - \frac{5 - 3x}{6} = \frac{x}{3} + 1$$