Factoring:

The number of positive factors of $n = p_1^{k_1} \cdots p_m^{k_m}$ is $\prod_{i=1}^{m}(k_i + 1)$

SFFT:

$xy + xk + yj + jk = (x + j)(y + k)$

(1) Find the number of positive integral pairs (x, y) that satisfy the equation $xy - x - y = 5$.

(2) Find the number of pairs of integers (x, y) that satisfy the equation $xy - x - y = 5$.

(3) Find the number of non-congruent rectangles with integer side lengths whose area is 5 more than its semiperimeter.

(4) Find the number of non-congruent rectangles with integer side lengths whose area is 5 more than the sum of its semiperimeter and one of its side lengths.

(5) Find all positive integral pairs (x, y) that satisfy the equation $xy + x - 2y = 13$.

(6) Find all positive integral pairs (x, y) that satisfy the equation $xy + 7x - 6y = 54$.

(7) Find all positive integral pairs (x, y) that satisfy the equation $xy - 6x - 5y + 18 = 0$.

(8) Penny's age is the *sum* of the ages of her two brothers. Six years ago, her age was the *product* of the ages of her two brothers. How old is Penny and her brothers?

(9) Two different prime numbers between 4 and 18 are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

(A) 22 **(B)** 60 **(C)** 119 **(D)** 180 **(E)** 231

(10) A rectangular floor measures a by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half of the area of the entire floor. How many possibilities are there for the ordered pair (a, b)?

(11) Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

(12) Let m, n be positive integers such that $\frac{1}{m} + \frac{1}{n} = \frac{19}{94}$. What is the value of $m + n$?

(13) Find $3x^2y^2$ if x and y are integers such that $y^2 + 3x^2y^2 = 30x^2 + 517$.

(14) How many non-congruent right triangles with positive integer leg lengths have areas that are numerically equal to 3 times their perimeters?