

Mini-math Div 3/4: Friday, September 27, 2024 (6.4-6.14) - 20 minutes

SOLUTIONS

1. (1 point) Suppose $V(x) = \int_0^{x^2} \sin t \, dt$. What is the derivative, $V'(x)$?

- A. $\cos x$ B. $\sin x$ C. $\sin x^2$ D. $2x \sin x^2$ E. $2x \cos x^2$

Solution: By FTC I,

$$V'(t) = \sin(x^2) \cdot \frac{d}{dx}(x^2) = 2x \sin(x^2)$$

(D) is correct.

2. (1 point) Given $\int_1^7 f(x) \, dx = 4$, $\int_{-1}^7 f(x) \, dx = -3$, and $\int_1^5 f(x) \, dx = 6$, find $\int_{-1}^5 (2f(x) + 3) \, dx$

- A. -2 B. 15 C. 16 D. 17 E. 28

Solution:

$$\int_{-1}^5 f(x) \, dx = \int_{-1}^7 f(x) \, dx - \int_1^7 f(x) \, dx + \int_1^5 f(x) \, dx = -3 - 4 + 6 = -1$$

so

$$\int_{-1}^5 (2f(x) + 3) \, dx = 2(-1) + \int_{-1}^5 3 \, dx = -2 + 18 = 16$$

(C) is correct.

3. (1 point) Using the substitution $u = x^3 - 2$, $\int_{-2}^3 x^2(x^3 - 2)^3 \, dx$ is equal to which of the following?

- A. $3 \int_{-10}^{25} u^3 \, du$ B. $\int_{-10}^{25} u^3 \, du$ C. $\frac{1}{3} \int_{-10}^{25} u^3 \, du$ D. $\int_{-2}^3 u^3 \, du$ E. $\frac{1}{3} \int_{-2}^3 u^3 \, du$

Solution: Changing the bounds of integration, $-2 \mapsto (-2)^3 - 2 = -8 - 2 = -10$ and $3 \mapsto 3^3 - 2 = 27 - 2 = 25$. Since $u = x^3 - 2$, we have $du = 3x^2 \, dx$, so $x^2 \, dx = \frac{1}{3} \, du$.

(C) is correct.

4. (1 point) $\int_0^1 \frac{2x-3}{x^2-5x+6} dx$ is

- A. $\ln\left(\frac{16}{27}\right)$ B. $\ln 8$ C. $\ln 27$ D. $\ln 432$ E. divergent

Solution:

$$\begin{aligned}\int_0^1 \frac{2x-3}{x^2-5x+6} dx &= \int_0^1 \frac{2x-3}{(x-2)(x-3)} dx = \int_0^1 \left(\frac{-1}{x-2} + \frac{3}{x-3} \right) dx \\ &= (-\ln|x-2| + 3\ln|x-3|) \Big|_0^1 = 4\ln 2 - 3\ln 3 = \ln \frac{16}{27}\end{aligned}$$

(A) is correct.

5. (1 point) $\int_1^\infty xe^{-x^2} dx$ is

- A. $-\frac{1}{e}$ B. $\frac{1}{2e}$ C. $\frac{1}{e}$ D. $\frac{2}{e}$ E. divergent

Solution:

$$\int_1^\infty xe^{-x^2} dx = \lim_{b \rightarrow \infty} \int_1^b xe^{-x^2} dx = \lim_{b \rightarrow \infty} -\frac{1}{2}e^{-x^2} \Big|_1^b = \lim_{b \rightarrow \infty} -\frac{1}{2}(e^{-b^2} - e^{-1}) = \frac{1}{2e}$$

(B) is correct.

6. (1 point) $\int_1^8 t^{-2/3} dt =$

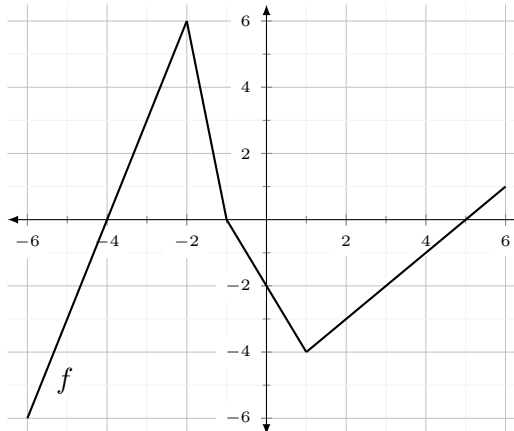
- A. -3 B. -1 C. $\frac{93}{160}$ D. 1 E. 3

Solution:

$$\int_1^8 t^{-2/3} dt = 3t^{1/3} \Big|_1^8 = 3(2 - 1) = 3$$

(E) is correct.

7. (1 point) To the right is a graph of the function $f(x)$. Suppose $g(x) = \int_a^x f(t) dt$ and $g(1) = 3$. What is the minimum value of $g(x)$ on $[-6, 2]$?



- A. -8
B. -5
C. -4
D. -3
E. -2

Solution: The minimum occurs at a point where $g' = f$ changes from negative to positive ($x = -4$) or at an endpoint. By the Net Change Theorem, $g(b) = g(1) + \int_1^b g'(t) dt = 3 + \int_1^b f(t) dt$. We test:

$$g(-6) = 3 + 4 - 9 + 6 = 4$$

$$g(-4) = 3 + 4 - 9 = -2$$

$$g(2) = 3 - 3.5 = 0.5$$

(E) is correct.

8. (2 points) Find $\int \frac{dx}{\sqrt{-x^2 + 4x - 3}}$

Solution:

$$\int \frac{dx}{\sqrt{-x^2 + 4x - 3}} = \int \frac{dx}{\sqrt{-(x-2)^2 + 4 - 3}} = \int \frac{dx}{\sqrt{1 - (x-2)^2}} = \arcsin(x-2) + C$$

9. (2 points) Find $\int (3x - 1) \sin x \, dx$

Solution: Use

$$\begin{aligned} f &= 3x - 1 & g' &= \sin x \, dx \\ f' &= 3 \, dx & g &= -\cos x \end{aligned}$$

so that

$$\int (3x - 1) \sin x \, dx = -(3x - 1) \cos x - \int 3(-\cos x) \, dx = (1 - 3x) \cos x + 3 \sin x + C$$