Systems of equations

A *system of equations* is a set of one or more equations involving some number of variables. The solution(s) to a system of equations are the values of the variables such that every equation is simultaneously satisfied. We have solved some basic systems of equations before, for instance:

Example 1.

$$\begin{aligned} \triangle + \blacklozenge + \blacklozenge &= 44 \\ \blacklozenge + \blacklozenge + \blacklozenge + \blacklozenge + \blacklozenge &= 70 \end{aligned}$$

Here, the second equation is only in terms of one variable, so we can easily solve by dividing by 5:

$$\blacklozenge = 70/5 = 14$$

Now we can us this in the first equation to get

$$\begin{aligned} \triangle + 14 \cdot 2 &= 44 \\ \triangle &= 44 - 28 = 16 \end{aligned}$$

(You should check that this works - indeed, $16 + 2 \cdot 14 = 44$ and $5 \cdot 14 = 70$.)

If none of the equations are in terms of a single variable, then we need to work a little harder. Here, we use the method of *elimination*, where we combine equations to *eliminate* a variable. The easiest case of this is when the number of one variable is the same in each equation.

Example 2.

$$\begin{aligned} \triangle + \triangle + \square &= 25 \\ \triangle + \triangle + \square + \square + \square &= 63 \end{aligned}$$

There is an equal number of $\triangle$ in each equation, so we can subtract the first equation from the second equation to get

$$\square + \square = 63 - 25 = 38$$

This looks like the first type of problem (Example 1) we solved, since now we have an equation only in terms of one variable. Dividing by 2,

$$\square = 38/2 = 19$$

Now we can us this in the first (or second) equation to get

$$\begin{aligned} \triangle + \triangle &= 25 - 19 = 6 \\ \triangle &= 3 \end{aligned}$$

(You should check that this works - indeed, $2 \cdot 3 + 19 = 25$ and $2 \cdot 3 + 3 \cdot 19 = 63$.)

You are now ready for Series 0.

What if none of the variables have the same count in the given equations? In that case, we can scale an equation first, in order to obtain equations that resemble Example 2.

Example 3. Note: we may omit the + sign between similar shapes to make the equation shorter.

$$\blacktriangle\,\blacktriangle + \square = 39$$
$$\blacktriangle\,\blacktriangle\,\blacktriangle\,\blacktriangle\,\blacktriangle\,\blacktriangle\,\blacktriangle\,\blacktriangle\,\blacktriangle\,\blacktriangle + \square\,\square\square = 165$$

We could match up the count of $\blacktriangle$ by multiplying the first equation by 5, but an easier choice would be to multiply the first equation by 3 to match up the count of $\square$:

$$\blacktriangle\,\blacktriangle\,\blacktriangle\,\blacktriangle\,\blacktriangle\,\blacktriangle + \square\,\square\square = 117$$

Make sure not to forget to multiply every term by 3 when you do this. Combining this to the 2nd given equation, we now have something similar to Example 2. In particular, subtracting from the 2nd given equation,

$$\blacktriangle\,\blacktriangle\,\blacktriangle\blacktriangle = 165 - 117 = 48$$
$$\blacktriangle = 48/4 = 12$$

Now we can us this in the first (or second) equation to get

$$12 \cdot 2 + \square = 39$$
$$\square = 39 - 24 = 15$$

(You should check that this works - indeed, $2 \cdot 12 + 15 = 39$ and $10 \cdot 12 + 3 \cdot 15 = 165$.)

You are now ready for Series 1.