Name: ______________________ Mark: ______ / 17

Mini-math Div 3/4: Monday, March 18, 2024 (10.10-10.15) - (25 minutes)

1. (3 points) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges to S. If S_n is used to approximate S, what is the least value of n for which the alternating series error bound guarantees an error to strictly within 0.01?

2. (3 points) Let $P(x)$ be the fifth-degree Taylor Polynomial for a function f about $x = 1$. Information about the maximum of the absolute value of selected derivatives of f over various intervals is given below.

$$\max_{0 \le x \le 1.5} |f^{(4)}(x)| = 4.6, \quad \max_{0 \le x \le 1.5} |f^{(5)}(x)| = 7.2, \quad \max_{0 \le x \le 1.5} |f^{(6)}(x)| = 6.8,$$
$$\max_{1 \le x \le 1.5} |f^{(4)}(x)| = 3.2, \quad \max_{1 \le x \le 1.5} |f^{(5)}(x)| = 4.7, \quad \max_{1 \le x \le 1.5} |f^{(6)}(x)| = 5.1$$

Find the smallest value of k for which the Lagrange error bound guarantees that

$$|f(1.5) - P(1.5)| \le k$$

3. (4 points) Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n2^n}$

4. (3 points) What is the Maclaurin series for $\frac{\cos x - 1}{x}$? Assume differentiability at 0 (e.g. the function has a value at 0 which makes it differentiable). You may, but are not required to, express your answer in summation notation.

5. (4 points) Let f be a function with $f(0) = 2$ and $f'(x) = \arctan x$. Write the first three non-zero terms of the Maclaurin series for f.