Mini-math Div 3/4: Friday, January 12, 2024 (9.1-9.6) - 20 minutes SOLUTIONS

1. (3 points) Write an equation for the line tangent to the curve defined by $r(t) = \langle 2^t, 1/t \rangle$ at the point where x = 8.

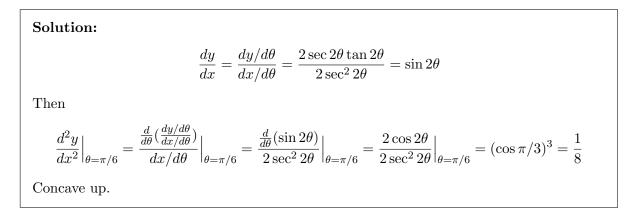
Solution: $2^t = 8$ gives t = 3. At this value, y(3) = 1/3. Now,

$$\frac{dy}{dx}\Big|_{t=3} = \frac{dy/dt}{dx/dt}\Big|_{t=3} = \frac{-t^{-2}}{2^t \ln 2}\Big|_{t=3} = \frac{-1/9}{8 \ln 2} = -\frac{1}{72 \ln 2}$$

by point-slope, an equation of the tangent line is

$$y - \frac{1}{3} = -\frac{1}{72\ln 2}(x - 8)$$

2. (4 points) If $x(\theta) = \tan 2\theta$ and $y(\theta) = \sec 2\theta$, find the concavity at $\theta = \pi/6$.



3. (2 points) Write down (but do not evaluate) an integral which represents the length of the curve described by the parametric equations $x = t^3/3$ and $y = t^2/2$ from t = 0 to t = 1. (Extra challenge: find the exact value.)

Solution: First, note that
$$x'(t) = t^2$$
 and $y'(t) = t$
$$L = \int_0^1 \sqrt{t^4 + t^2} dt \quad \left(= \frac{(t^2 + 1)^{3/2}}{3} \Big|_0^1 = \frac{2\sqrt{2} - 1}{3} \right)$$

4. (3 points) If f is a vector-valued function defined by $f(t) = \langle 2 \sin t, \cos 2t \rangle$, then what is $f''(\pi/3)$?

Sol	ution:

Solution:

 $f'(t) = \langle 2\cos t, -2\sin 2t \rangle,$ $f''(t) = \langle -2\sin t, -4\cos 2t \rangle,$ $f''(\pi/3) = \langle -\sqrt{3}, 2 \rangle$

5. (3 points) Find the vector-valued function f(t) that satisfies the initial conditions $f(1) = \langle 4, 5 \rangle$, and $f'(t) = \langle 6t, 7 \rangle$.

$$f(t) = \langle 4 + \int_{1}^{t} 6u \, du, 5 + \int_{1}^{t} 7 \, du \rangle$$
$$= \langle 4 + 3(t^{2} - 1), 5 + 7(t - 1) \rangle$$

6. (4 points) (Calculator-active) At time $t \ge 0$, a particle moving in the *xy*-plane has velocity vector given by $v(t) = \langle \sin(t^2), 2^{\sqrt{t}} \rangle$. If the particle is at point (-3, 1) at time t = 0, how far is the particle from the origin at time t = 3?

Solution: We calculate

$$\begin{aligned} x(3) &= -3 + \int_0^3 \sin(t^2) \, dt \approx -2.22644 \\ y(3) &= 1 + \int_0^3 2^{\sqrt{t}} \, dt \approx 7.93628 \end{aligned}$$

Then

$$d = \sqrt{[x(3)]^2 + [y(3)]^2} \approx 8.243$$