Mini-math Div 3/4: Monday, December 4, 2023 (8.7-8.13) (20 minutes)

SOLUTIONS

1. In this question, you do not need to simplify your answer. Find an integral (but do not evaluate) that represents the volume of the solid whose base is the region R bounded by $y = x/2$ and $y = \sqrt{2x}$, if:

   (a) (2 points) cross-sections perpendicular to the x-axis are rectangles whose heights are twice their base.

   Solution:
   $$\int_0^8 2(\sqrt{2x} - x/2)^2\, dx$$

   (b) (2 points) cross-sections perpendicular to the x-axis are right isosceles triangles whose hypotenuse lies on the base.

   Solution:
   $$\int_0^8 \frac{1}{4}(\sqrt{2x} - x/2)^2\, dx$$

   (c) (2 points) cross-sections perpendicular to the y-axis are semi-circles.

   Solution:
   $$\int_0^4 \frac{\pi}{2}\left(\frac{2y - y^2/2}{2}\right)^2 dy = \int_0^4 \frac{\pi}{8}\left(2y - y^2/2\right)^2\, dy$$

   (d) (2 points) cross-sections perpendicular to the y-axis are right isosceles triangles whose hypotenuse does not lie on the base.

   Solution:
   $$\int_0^4 \frac{1}{2}\left(2y - y^2/2\right)^2\, dy$$

2. In this question, you do not need to simplify your answer. Consider the region R bounded by $y = x/2$ and $y = \sqrt{2x}$.

   (a) Find an integral (but do not evaluate) that represents the volume of the solid of revolution if we revolve the region R:

      i. (2 points) about the x-axis.

      Solution:

      $$\pi \int_0^8 \left[\sqrt{2x}^2 - (x/2)^2\right] dx = \pi \int_0^8 \left(2x - \frac{x^2}{4}\right) dx$$

      ii. (2 points) about the y-axis.

      Solution:

      $$\pi \int_0^4 \left[(2y)^2 - (y^2/2)^2\right] dy$$

      iii. (2 points) about the line $y = -1$.

      Solution:

      $$\pi \int_0^8 \left[(\sqrt{2x} + 1)^2 - (x/2 + 1)^2\right] dx$$

      iv. (2 points) about the line $x = 10$.

      Solution:

      $$\pi \int_0^4 \left[(10 - y^2/2)^2 - (10 - 2y)^2\right] dy$$

   (b) Find an integral (but do not evaluate) that represents the perimeter of the region R.

   Solution:

   $$\int_0^8 \sqrt{1 + \left(\frac{1}{2}\right)^2}\, dx + \int_0^8 \sqrt{1 + \left(\frac{1}{\sqrt{2x}}\right)^2}\, dx \quad \text{or} \quad 4\sqrt{5} + \int_0^8 \sqrt{1 + \left(\frac{1}{\sqrt{2x}}\right)^2}\, dx$$