Name: ______________________________ Mark: _______ / 14

Mini-math Div 3/4: Monday, October 23, 2023 (20 minutes)

1. (2 points) Solve the following differential equation:

$$\frac{dy}{dx} = xy\sin(x^2) \cdot \ln y$$

2. (2 points) During a chemical reaction, the rate of change of the amount of the chemical remaining is proportional to the amount remaining. At time $t = 0$, the amount of the chemical is 60 g. At time $t = 8$, the amount of the chemical is 12 g. At what time t is the amount of the chemical 4 g?

A. $\frac{4\sqrt{42}}{3}$ B. $\frac{28}{3}$ C. $\frac{8\ln 15}{\ln 5}$ D. $\frac{8\ln 4}{\ln 12}$ E. $\frac{8}{3}$

3. (2 points) Solve the following initial value problem:

$$\frac{dy}{dx} = y^2, \quad y(3) = -2$$

A. $y = \dfrac{1}{5/2 - x}$ for $x \neq 5/2$

B. $y = \dfrac{2}{5 - 2x}$ for $x > 5/2$

C. $y = -\dfrac{1}{x} - \dfrac{5}{3}$ for $x \neq 0$

D. $y = -\dfrac{5x + 3}{3x}$ for $x > 0$

4. The number of squirrels in a park at time t is modelled by the function $y = F(t)$ that satisfies the logistic differential equation $\frac{dy}{dt} = \frac{1}{2000}y(1500 - y)$, where $t \geq 0$ is measured in weeks. The number of squirrels in the park at time $t = 0$ is $F(0) = b$, where b is a positive constant.

   (a) i. (1 point) If $b = 300$, what is the largest rate of increase in the number of squirrels in the park?

   ii. (1 point) If $b = 1000$, what is the largest rate of increase in the number of squirrels in the park?

(b) (2 points) If $b = 150$, find $\lim_{t \to \infty} F(t)$ and interpret the meaning of this limit in the context of the problem. For reference, the differential equation is $\frac{dy}{dt} = \frac{1}{2000} y(1500 - y)$.

(c) (4 points) (*) Find the function $F(t)$ if $b = 500$.