Mini-math Div 3/4: Wednesday, October 18, 2023 (7.1-7.5) - 18 minutes

SOLUTIONS

1. Write a differential equation that describes the following relationships. If necessary, use k as the constant of proportionality.

   (a) (2 points) The rate of change of population, P, with respect to time, t, is inversely proportional to the square root of time and directly proportional to the area, A, that the population covers.

   > **Solution:**
   >
   > $$\frac{dP}{dt} = \frac{kA}{\sqrt{t}}$$

   (b) (2 points) The position of a particle is given by $s(t)$, where t is measured in seconds. Its acceleration is directly proportional to its position. When the particle is at position 4 units, its acceleration is 2 units/s^2.

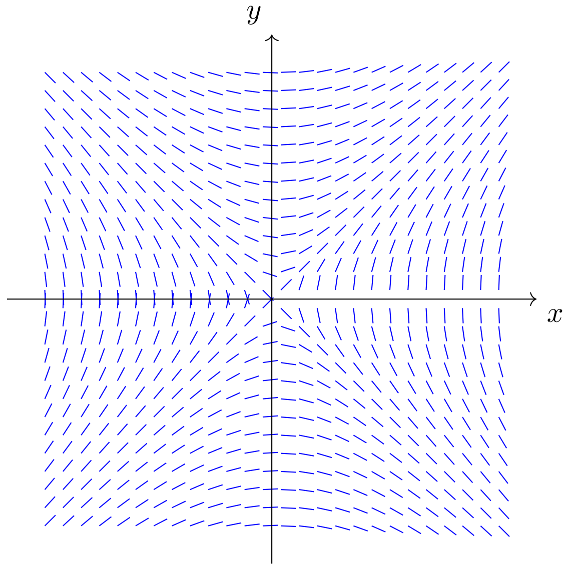
   > **Solution:**
   >
   > $$\frac{d^2s}{dt^2} = \frac{1}{2}s$$

2. (4 points) Determine the value of k, if any, for which $y = \sin(2x) - k\sin(4x)$ would be a solution to the differential equation $y'' + 4y = 3\sin(4x)$.

> **Solution:**
>
> $$y' = 2\cos(2x) - 4k\cos(4x)$$
> $$y'' = -4\sin(2x) + 16k\sin(4x)$$
>
> Then
>
> $$[-4\sin(2x) + 16k\sin(4x)] + 4[\sin(2x) - k\sin(4x)] = 3\sin(4x)$$
>
> so $16k - 4k = 3$, giving $k = 1/4$.

3. (2 points) What differential equation can the slope field to the right represent?

   A. $\frac{dy}{dx} = -x/y$
   B. $\frac{dy}{dx} = -y/x$
   C. $\frac{dy}{dx} = y^2$
   D. $\frac{dy}{dx} = x/y$
   E. $\frac{dy}{dx} = y/x$

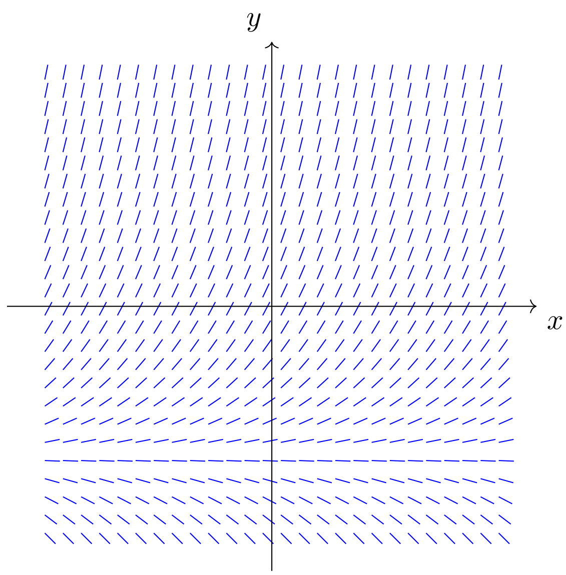


Solution:

At $x = 0$, the slopes are 0, so (A) or (D). Since the slopes are positive for $x > 0$ and $y < 0$, (D) is the answer.

4. (2 points) The slope field for a certain differential equation is shown to the right. Which of the following could be a particular solution to the differential equation?

   A. $y = x^3$
   B. $y = \frac{1}{x+2}$
   C. $y = -2^x - 2$
   D. $y = e^{-x} - 2$
   E. $y = e^x + 2$



Solution: The solutions have a horizontal asymptote as $x \to -\infty$, so (C) is the answer.

5. Consider the initial value problem $\frac{dy}{dx} = 2x + y$ and $y(1) = 2$.

(a) (2 points) Find an approximation of $y(1.2)$ using Euler's Method with two equal steps.

Solution:

$y(1.1) \approx y(1) + y'(1)(1.1 - 1) = 2 + (2(1) + 2)(0.1) = 2.4,$
$y(1.2) \approx y(1.1) + y'(1.1)(1.2 - 1.1) \approx 2.4 + (2(1.1) + 2.4)(1.2 - 1.1) = 2.4 + (4.6)(0.1) = 2.86$

(b) (2 points) Is your estimate in part (a) an overestimate or an underestimate?

Solution: At $(1, 2)$,

$$\frac{d^2y}{dx^2} = 2 + \frac{dy}{dx} = 2 + 2x + y > 0$$

so the function is concave up. Therefore, the estimate will be an underestimate. (Aside: the actual value is about 2.92842. If we used 10 equal steps, we would get the approximation 2.91397)