

Mini-math Div 3/4: Friday, September 15, 2022 (18 minutes)

SOLUTIONS

1. (1 point) Given  $\int_1^7 f(x) dx = 4$ ,  $\int_{-1}^7 f(x) dx = -3$ , and  $\int_1^5 f(x) dx = 6$ , find  $\int_{-1}^5 (2f(x) + 3) dx$
- A. -2                      B. 15                      C. 16                      D. 17                      E. 28

**Solution:**

$$\int_{-1}^5 f(x) dx = \int_{-1}^7 f(x) dx - \int_1^7 f(x) dx + \int_1^5 f(x) dx = -3 - 4 + 6 = -1$$

so

$$\int_{-1}^5 (2f(x) + 3) dx = 2(-1) + \int_{-1}^5 3 dx = -2 + 18 = 16$$

(C) is correct.

2. (1 point) Using the substitution  $u = x^3 - 2$ ,  $\int_{-2}^3 x^2(x^3 - 2)^3 dx$  is equal to which of the following?

- A.  $3 \int_{-10}^{25} u^3 du$   
B.  $\int_{-10}^{25} u^3 du$   
C.  $\frac{1}{3} \int_{-10}^{25} u^3 du$   
D.  $\int_{-2}^3 u^3 du$   
E.  $\frac{1}{3} \int_{-2}^3 u^3 du$

**Solution:** Changing the bounds of integration,  $-2 \mapsto (-2)^3 - 2 = -8 - 2 = -10$  and  $3 \mapsto 3^3 - 2 = 27 - 2 = 25$ . Since  $u = x^3 - 2$ , we have  $du = 3x^2 dx$ , so  $x^2 dx = \frac{1}{3} du$ .

(C) is correct.

3. (1 point)  $\int_0^1 \frac{2x-3}{x^2-5x+6} dx$  is

- A.  $\ln\left(\frac{16}{27}\right)$       B.  $\ln 8$       C.  $\ln 27$       D.  $\ln 432$       E. divergent

**Solution:**

$$\begin{aligned}\int_0^1 \frac{2x-3}{x^2-5x+6} dx &= \int_0^1 \frac{2x-3}{(x-2)(x-3)} dx = \int_0^1 \left( \frac{-1}{x-2} + \frac{3}{x-3} \right) dx \\ &= (-\ln|x-2| + 3\ln|x-3|) \Big|_0^1 = 4\ln 2 - 3\ln 3 = \ln \frac{16}{27}\end{aligned}$$

(A) is correct.

4. (1 point)  $\int_1^\infty xe^{-x^2} dx$  is

- A.  $-\frac{1}{e}$       B.  $\frac{1}{2e}$       C.  $\frac{1}{e}$       D.  $\frac{2}{e}$       E. divergent

**Solution:**

$$\int_1^\infty xe^{-x^2} dx = \lim_{b \rightarrow \infty} \int_1^b xe^{-x^2} dx = \lim_{b \rightarrow \infty} -\frac{1}{2}e^{-x^2} \Big|_1^b = \lim_{b \rightarrow \infty} -\frac{1}{2}(e^{-b^2} - e^{-1}) = \frac{1}{2e}$$

(B) is correct.

5. (1 point)  $\int_1^8 t^{-2/3} dt =$

- A. -3                      B. -1                      C.  $\frac{93}{160}$                       D. 1                      E. 3

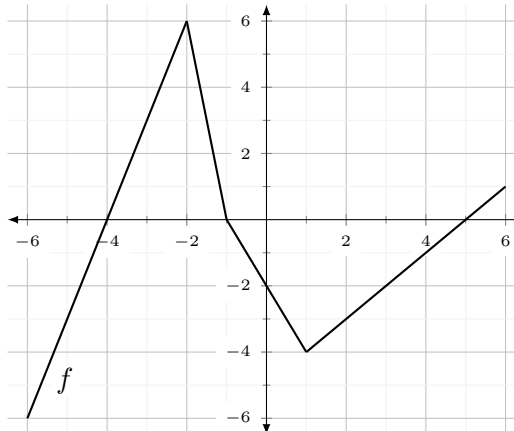
**Solution:**

$$\int_1^8 t^{-2/3} dt = 3t^{1/3} \Big|_1^8 = 3(2 - 1) = 3$$

(E) is correct.

6. (1 point) To the right is a graph of the function  $f(x)$ . Suppose  $g(x) = \int_a^x f(t) dt$  and  $g(1) = 3$ . What is the minimum value of  $g(x)$  on  $[-6, 2]$ ?

- A. -8  
B. -5  
C. -4  
D. -3  
E. -2



**Solution:** The minimum occurs at a point where  $g' = f$  changes from negative to positive ( $x = -4$ ) or at an endpoint. By the Net Change Theorem,  $g(b) = g(1) + \int_1^b g'(t) dt = 3 + \int_1^b f(t) dt$ . We test:

$$g(-6) = 3 + 4 - 9 + 6 = 4$$

$$g(-4) = 3 + 4 - 9 = -2$$

$$g(2) = 3 - 3.5 = 0.5$$

(E) is correct.

7. (2 points) Find  $\int \frac{dx}{\sqrt{-x^2 + 4x - 3}}$

**Solution:**

$$\int \frac{dx}{\sqrt{-x^2 + 4x - 3}} = \int \frac{dx}{\sqrt{-(x-2)^2 + 4 - 3}} = \int \frac{dx}{\sqrt{1 - (x-2)^2}} = \arcsin(x-2) + C$$

8. (2 points) Find  $\int (3x - 1) \sin x \, dx$

**Solution:** Use

$$\begin{aligned} f &= 3x - 1 & g' &= \sin x \, dx \\ f' &= 3 \, dx & g &= -\cos x \end{aligned}$$

so that

$$\int (3x - 1) \sin x \, dx = -(3x - 1) \cos x - \int 3(-\cos x) \, dx = (1 - 3x) \cos x + 3 \sin x + C$$