Numerical Integration

1. Consider the following table of values of $f(x)$:

x	0	1	2	3	4	5	6	7	8
$f(x)$	5	1	3	2	4	7	9	10	9

Approximate $\displaystyle\int_0^8 f(x)\,dx$ using the stated method.

(a) Right Riemann sum with 8 equal subintervals

Solution:

$$\begin{aligned}&f(1)\cdot 1+f(2)\cdot 1+f(3)\cdot 1+f(4)\cdot 1+f(5)\cdot 1+f(6)\cdot 1+f(7)\cdot 1+f(8)\cdot 1\\&\quad =1+3+2+4+7+9+10+9=45\end{aligned}$$

(b) Left Riemann sum with 8 equal subintervals

Solution:

$$\begin{aligned}&f(0)\cdot 1+f(1)\cdot 1+f(2)\cdot 1+f(3)\cdot 1+f(4)\cdot 1+f(5)\cdot 1+f(6)\cdot 1+f(7)\cdot 1\\&\quad =5+1+3+2+4+7+9+10=41\end{aligned}$$

(c) Trapezoid Rule with 8 equal subintervals

Solution:

$$\begin{aligned}&\left(\frac{f(0)+f(1)}{2}\right)\cdot 1+\left(\frac{f(1)+f(2)}{2}\right)\cdot 1+\left(\frac{f(2)+f(3)}{2}\right)\cdot 1+\left(\frac{f(3)+f(4)}{2}\right)\cdot 1\\&\quad +\left(\frac{f(4)+f(5)}{2}\right)\cdot 1+\left(\frac{f(5)+f(6)}{2}\right)\cdot 1+\left(\frac{f(6)+f(7)}{2}\right)\cdot 1+\left(\frac{f(7)+f(8)}{2}\right)\cdot 1\\&=3+2+5/2+3+11/2+8+19/2+19/2=43\end{aligned}$$

(d) Right Riemann sum with 4 equal subintervals

Solution:

$$f(2)\cdot 2+f(4)\cdot 2+f(6)\cdot 2+f(8)\cdot 2=6+8+18+18=50$$

(e) Left Riemann sum with 4 equal subintervals

Solution:

$$f(0) \cdot 2 + f(2) \cdot 2 + f(4) \cdot 2 + f(6) \cdot 2 = 10 + 6 + 8 + 18 = 42$$

(f) Midpoint Rule with 4 equal subintervals

Solution:

$$f(1) \cdot 2 + f(3) \cdot 2 + f(5) \cdot 2 + f(7) \cdot 2 = 2 + 4 + 14 + 20 = 40$$

(g) Trapezoid Rule with 4 equal subintervals

Solution:

$$\left(\frac{f(0) + f(2)}{2}\right) \cdot 2 + \left(\frac{f(2) + f(4)}{2}\right) \cdot 2 + \left(\frac{f(4) + f(6)}{2}\right) \cdot 2 + \left(\frac{f(6) + f(8)}{2}\right) \cdot 2$$
$$= 8 + 7 + 13 + 18 = 46$$

(h) Right Riemann sum with 2 equal subintervals

Solution: $f(4) \cdot 4 + f(8) \cdot 4 = 16 + 36 = 52$

(i) Left Riemann sum with 2 equal subintervals

Solution: $f(0) \cdot 4 + f(4) \cdot 4 = 20 + 16 = 36$

(j) Midpoint Rule with 2 equal subintervals

Solution: $f(2) \cdot 4 + f(6) \cdot 4 = 12 + 36 = 48$

(k) Trapezoid Rule with 2 equal subintervals

Solution: $\left(\frac{f(0)+f(4)}{2}\right) \cdot 4 + \left(\frac{f(4)+f(8)}{2}\right) \cdot 4 = 18 + 26 = 44$

2. Consider the following table of values of $f(x)$:

x	0	2	3	6	9	11	15
$f(x)$	4	1	2	-2	4	6	10

Approximate $\int_0^{15} f(x)\,dx$ using the stated method.

(a) Right Riemann Sum with 6 intervals

Solution:

$$\begin{aligned}&f(2)\cdot 2 + f(3)\cdot 1 + f(6)\cdot 3 + f(9)\cdot 3 + f(11)\cdot 2 + f(15)\cdot 4\\ &= 2+2-6+12+12+40 = 62\end{aligned}$$

(b) Left Riemann Sum with 6 intervals

Solution:

$$\begin{aligned}&f(0)\cdot 2 + f(2)\cdot 1 + f(3)\cdot 3 + f(6)\cdot 3 + f(9)\cdot 2 + f(11)\cdot 4\\ &= 8+1+6-6+8+24 = 41\end{aligned}$$

(c) Trapezoid Rule with 6 intervals

Solution:

$$\begin{aligned}&\left(\frac{f(0)+f(2)}{2}\right)\cdot 2 + \left(\frac{f(2)+f(3)}{2}\right)\cdot 1 + \left(\frac{f(3)+f(6)}{2}\right)\cdot 3\\ &+ \left(\frac{f(6)+f(9)}{2}\right)\cdot 3 + \left(\frac{f(9)+f(11)}{2}\right)\cdot 2 + \left(\frac{f(11)+f(15)}{2}\right)\cdot 4\\ &= 5 + 3/2 + 0 + 3 + 10 + 32 = 51.5\end{aligned}$$

(d) Midpoint Rule with 3 intervals

Solution:

$$\begin{aligned}&f(2)\cdot(3-0) + f(6)\cdot(9-3) + f(11)\cdot(15-9)\\ &= 3 - 12 + 36 = 27\end{aligned}$$