Student handouts with answers:

Mind Your P's and Two's

Recall the definition of the definite integral as a limit of a Riemann sum:

$$\int_a^b f(x)dx = \lim_{x\to\infty}\sum_{i=1}^{n} f\left(a+\Delta x\cdot i\right)\Delta x \text{ , where } \Delta x = \frac{b-a}{n} \text{ .}$$

In the chart below, there is an error in the expression for the limit of the Riemann sum: one of the numbers needs to be replaced with a 2. Circle the error and replace with a 2, explaining why the 2 should be there.

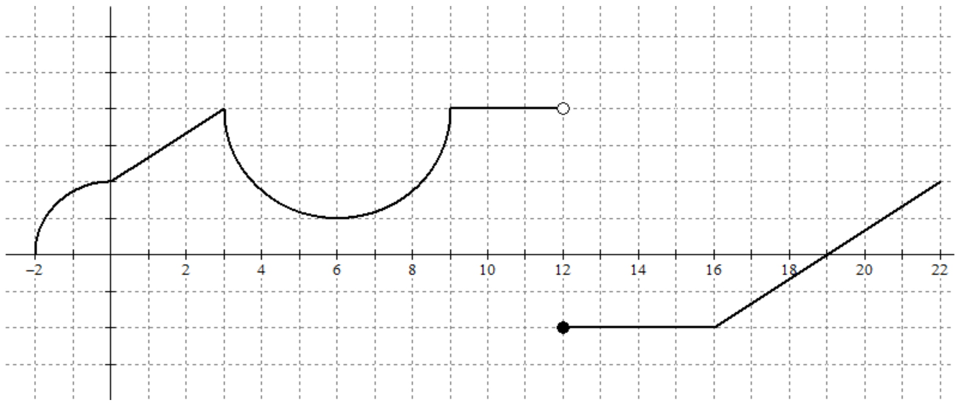
Definite Integral	Limit of Riemann sum	Explanation
$\int_4^6 \left(3x^2+5\right)dx$	$\lim_{n\to\infty}\sum_{i=1}^{n}\left[3\left(4+\frac{2i}{n}\right)^2+5\right]\frac{6}{n}$	Answer: The width of each interval, Δx , is given by $\frac{b-a}{n}$, so in this case, $\frac{6-4}{n}=\frac{2}{n}$.
$\int_2^7 (4x-1)dx$	$\lim_{n\to\infty}\sum_{i=1}^{n}\left[4\left(4+\frac{5i}{n}\right)^2-1\right]\frac{5}{n}$	Answer: The value of a is 2 and the height of the rectangle is given by $f\left(a+\Delta x\cdot i\right)$.
$\int_3^4 \left(6x^2+3x\right)dx$	$\lim_{n\to\infty}\sum_{i=1}^{n}\left[6\left(3+\frac{1i}{n}\right)^3+3\left(3+\frac{1i}{n}\right)\right]\frac{1}{n}$	Answer: The exponent in the integrand is a 2, not a 3.
$\int_5^9 (8-2x)dx$	$\lim_{n\to\infty}\sum_{i=1}^{n}\left[8-4\left(5+\frac{4i}{n}\right)\right]\frac{4}{n}$	Answer: The coefficient in the integrand is a 2, not a 4.
$\int_{-1}^{1} 3x^3dx$	$\lim_{n\to\infty}\sum_{i=1}^{n}\left[3\left(-1+\frac{1i}{n}\right)^3\right]\frac{2}{n}$	Answer: The value of Δx is $\frac{2}{n}$ and the height of the rectangle is given by $f\left(a+\Delta x\cdot i\right)$, so that 1 in the numerator should be replaced with a 2.

Apply Your Understanding of Summation Notation

In the chart below, the limit of a Riemann sum has been provided for you. Write the corresponding definite integral.

	Definite Integral	Limit of Riemann Sum
1.	Answer: $\int_0^6 \sqrt{2x+1}\,dx$	$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\sqrt{2\left(\frac{6}{n}\right)+1} \right] \frac{6}{n}$
2.	Answer: $\int_{-2}^{3} x^2 - 3\,dx$	$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-2+\frac{5i}{n}\right)^2 - 3 \right] \frac{5}{n}$
3.	Answer: $\int_1^6 3x - 4\,dx$	$\lim_{n\to\infty} \sum_{i=1}^{n} \left[3\left(1+\frac{5i}{n}\right) - 4 \right] \frac{5}{n}$
4.	Answer: $\int_{-2}^{4} x^3\,dx$	$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-2+\frac{6i}{n}\right)^3 \right] \frac{6}{n}$
5.	Answer: $\int_{-2}^{0} \sqrt{x^2+1}\,dx$	$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\sqrt{\left(-2+\frac{2i}{n}\right)^2 + 1} \right] \frac{2}{n}$
6.	Answer: $\int_2^6 5x + 7\,dx$	$\lim_{n\to\infty} \sum_{i=1}^{n} \left[5\left(2+\frac{4i}{n}\right) + 7 \right] \frac{4}{n}$
7.	Answer: $\int_0^4 6x^2 - 2\,dx$	$\lim_{n\to\infty} \sum_{i=1}^{n} \left[6\left(\frac{4i}{n}\right)^2 - 2 \right] \frac{4}{n}$
8.	Answer: $\int_1^3 4x^3 - 1\,dx$	$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{2i}{n}\right)^3 - 1 \right] \frac{2}{n}$

Translating Notation and Finding Definite Integral Values



The graph above consists of a quarter circle, a half circle and four line segments. For each of the expressions below, fill in the missing definite integrals. Then determine the value of each definite integral using geometric formulas (without using a calculator).

Limit of Riemann Sum	Definite Integral	Value of Definite Integral
$\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}(-2)\left(\frac{4}{n}\right)$	Answer: $\int_{12}^{16}(-2)\,dx$	Answer: -8
$\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(4-\sqrt{9-\left(\left(3+\frac{6i}{n}\right)-6\right)^2}\right)\left(\frac{6}{n}\right)$	Answer: $\int_{3}^{9}4-\sqrt{9-(x-6)^2}\,dx$	Answer: $24-\frac{9\pi}{2}$
$\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(\frac{2}{3}\left(\frac{3i}{n}\right)+2\right)\left(\frac{3}{n}\right)$	Answer: $\int_{0}^{3}\left(\frac{2}{3}x+2\right)dx$	Answer: 9
$\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(\sqrt{4-\left(-2+\frac{2i}{n}\right)^2}\right)\left(\frac{2}{n}\right)$	Answer: $\int_{-2}^{0}\sqrt{4-x^2}\,dx$	Answer: π
$\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}(4)\left(\frac{3}{n}\right)$	Answer: $\int_{9}^{12}4\,dx$	Answer: 12
$\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(\frac{2}{3}\left(\left(16+\frac{6i}{n}\right)-19\right)\right)\left(\frac{6}{n}\right)$	Answer: $\int_{16}^{22}\frac{2}{3}(x-19)\,dx$	Answer: 0