Volumes

1. (Calculator permitted) Let R be the region bounded by the curves $y = \sqrt{1 + x^3}$ and $y = x+1$. Set up integrals for the following, but do not evaluate.

   (a) The area of R

   (b) The volume of a solid whose base is R and cross-sections perpendicular to the x-axis are squares

   (c) The volume of a solid whose base is R and cross-sections perpendicular to the y-axis are equilateral triangles

   (d) The volume of the solid of revolution obtained by revolving R about the x-axis

2. (Calculator permitted) Let R be the region bounded by the curves $y = e^x$ and $y = 2x + 1$.

   (a) Find the area of R.

   (b) Find the volume of the solid of revolution obtained by revolving R about the x-axis.

   (c) Find the volume of the solid of revolution obtained by revolving R about the y-axis.

   (d) Find the volume of the solid of revolution obtained by revolving R about the line $y = 4$.

3. (Calculator permitted, but as a challenge you may try without a calculator) Let R be the region bounded by the curves $y = x + 1$, $y = \frac{x}{2} + 1$, and $y = 4 - x$.

   (a) Find the area of R.

   (b) Find the volume of the solid of revolution obtained by revolving R about the x-axis.

(c) Find the volume of the solid of revolution obtained by revolving R about the y-axis.

(d) (*) There exists a real number k such that if we revolve R about the line $x = k$, the resulting solid has the same volume as the solid obtained by revolving R about the x-axis. Find k.