CALCULUS AB
WORKSHEET ON INTEGRATION BY LONG DIVISION AND COMPLETING THE SQUARE

NAME _____
PERIOD _____

Use algebraic **long division** to solve the following integrals:

1. $\int \frac{x+1}{x-1}\,dx$
2. $\int \frac{x^3}{x^2+1}\,dx$
3. $\int_0^1 \frac{x^2}{x+1}\,dx$
4. $\int \frac{5x^2}{x^2+1}\,dx$
5. $\int \frac{2x^2-4}{x+1}\,dx$
6. $\int_2^3 \frac{y+1}{y-1}\,dy$
7. $\int \frac{x^3+x}{x-1}\,dx$
8. $\int_{-2}^1 \frac{x^3+3x-4}{x-2}\,dx$
9. $\int_0^2 \frac{2x^3+3x^2-17x-27}{x^2-9}\,dx$

Use the **completing the square** method to solve the following integrals:

10. $\int \frac{1}{t^2-10t+32}\,dt$
11. $\int_{-3}^{-1} \frac{1}{\sqrt{7-x^2-6x}}\,dx$
12. $\int \frac{4}{t^2-4t+20}\,dt$
13. $\int_{-1}^0 \frac{1}{\sqrt{3-x^2-2x}}\,dx$
14. $\int \frac{3}{x^2-6x+18}\,dx$
15. $\int \frac{dx}{x^2-4x+7}$
16. $\int_{3/2}^{9/4} \frac{1}{\sqrt{3x-x^2}}\,dx$
17. $\int_2^3 \frac{2x-3}{\sqrt{4x-x^2}}\,dx$
18. $\int \frac{2x}{x^2+6x+13}\,dx$

ANSWERS:

Long division: 1. $x + 2\ln|x-1| + C$ 2. $\frac{1}{2}x^2 - \frac{1}{2}\ln|x^2+1| + C$ 3. $\ln 2 - \frac{1}{2}$ 4. $5x - 5\tan^{-1}(x) + C$
5. $x^2 - 2x - 2\ln|x+1| + C$ 6. $1 + 2\ln 2$ 7. $\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\ln|x-1| + C$ 8. $21 - 20\ln 2$ 9. $10 + \ln\left(\frac{\sqrt{5}}{3}\right)$

Completing the square: 10. $\frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{t-5}{\sqrt{7}}\right) + C$ 11. $\frac{\pi}{6}$ 12. $\tan^{-1}\left(\frac{t-2}{4}\right) + C$ 13. $\frac{\pi}{6}$
14. $\tan^{-1}\left(\frac{x-3}{3}\right) + C$ 15. $\frac{1}{\sqrt{3}}\arctan\left(\frac{x-2}{\sqrt{3}}\right) + C$ 16. $\frac{\pi}{6}$ 17. $4 - 2\sqrt{3} + \frac{\pi}{6}$
18. $\ln|x^2+6x+13| - 3\arctan\left(\frac{x+3}{2}\right) + C$